

SC705: Advanced Statistics
Instructor: Natasha Sarkisian
Class notes: Multigroup SEM models

So far, we discussed analyses working with single samples. Often, however, we are interested to know whether the model (measurement model, structural model, or both) is the same across particular groups. Such groups may be defined by any dichotomous or ordinal variables in your data (e.g. gender, race/ethnicity, religious affiliation, etc.).

If both measurement and structural models are of interest, the analysis proceeds in stages: first, the equivalence of the measurement model is established, and second, the structural models are compared. Usually such analyses focus on making cross-group comparisons of factor loadings λ , structural paths γ and β , and the elements of the variance/covariance matrix for exogenous variables, Φ . In contrast, variances/covariances for errors and disturbance terms are rarely of interest in such analyses.

The logic of multigroup comparisons (for either measurement or structural model):

- Step 1. Estimate separate baseline models for each group to determine the best fitting model for each group
- Step 2. Estimate a joint yet unconstrained model for all groups (i.e. coefficients are allowed to vary freely across groups)
- Step 3. Estimate a joint constrained model where the parameters across groups are constrained to be equal to each other
- Step 4. Compare the fit of such constrained model with that of an unconstrained one using the difference in chi-square statistic. If the chi-square difference statistic does not reveal a significant difference between the unconstrained and constrained models, then we conclude that the model applies across groups. If the chi-square statistic does reveal a significant difference between the unconstrained and constrained model, then we can use modification indices to partially remove the constraints (i.e., pinpoint where the differences are).

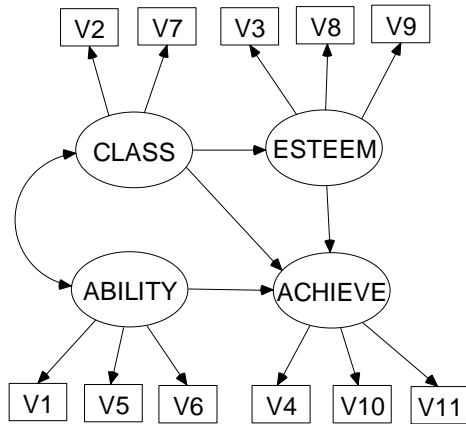
Sometimes, the groups can also be multiple samples from the same population – that case is known as model cross-validation, and its goal is to establish whether the model developed for one sample holds for another sample. The procedure is quite similar, except here we only estimate a baseline model for our original sample and then apply it to the validation sample to see if it holds.

1. Measurement model.

The first step in multigroup analysis would be to evaluate the assumption of measurement model invariance. Measurement invariance may be defined with varying degrees of stringency, depending on which parameters are constrained to be equal. Sometimes, we may only test for invariance on number of factors; most commonly, however, we focus on identifying whether the factor loadings are invariant. Further, if we focus only on the measurement model, we will also test for invariant variances/covariances among the latent factors. While it is possible also to test for equality of error variances and covariances across groups, it's usually considered excessively

stringent. So in practice, it is common to define measurement invariance as invariance of the factor loadings.

Let's continue with the example we started earlier, and add another group. Suppose the data we used in the last example are the data for boys, and we have similar data for girls:



Boys (N=414):

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
X1	1.00										
X2	.345	1.00									
X3	.287	.377	1.00								
X4	.252	.579	.114	1.00							
X5	.637	.335	.253	.255	1.00						
X6	.768	.339	.302	.278	.600	1.00					
X7	.254	.703	.337	.591	.312	.313	1.00				
X8	.166	.429	.724	.143	.264	.216	.364	1.00			
X9	.104	.411	.506	.142	.199	.163	.551	.630	1.00		
X10	.247	.601	.202	.880	.288	.309	.621	.290	.199	1.00	
X11	.208	.526	.127	.827	.253	.231	.676	.155	.234	.808	1.00

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
SD	13.9	13.4	10.0	15.9	9.4	4.9	5.9	11.1	5.9	11.8	7.9

Girls (N=412):

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
X1	1.00										
X2	.293	1.00									
X3	.278	.377	1.00								
X4	.109	.379	-.07	1.00							
X5	.656	.300	.211	.129	1.00						
X6	.825	.331	.311	.160	.677	1.00					
X7	.191	.659	.336	.378	.137	.263	1.00				
X8	.146	.400	.677	.042	.185	.211	.409	1.00			
X9	.168	.474	.554	.026	.140	.233	.626	.657	1.00		
X10	.123	.406	.026	.857	.186	.185	.408	.071	.049	1.00	
X11	.079	.362	.039	.822	.082	.124	.509	-.03	.080	.806	1.00

	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11
SD	14.5	11.0	9.5	16.3	8.9	5.0	4.9	10.7	5.3	11.1	7.1

We have already examined the model for boys in prior example – that would be our baseline model for boys. Typically we would want to focus on achieving good fit for that model, and do the same for a baseline model for girls. For brevity here we will start right away with estimating a joint unconstrained model. Note, that to ensure that the models are similar to those estimated as baseline, when estimating this unconstrained model we could have also specified approximate starting values for various parameters based on the baseline models (for that, we could use ST command to give the starting values and specify NS option in OU statement to suppress the automatic starting values).

Boys and girls combined, unconstrained:

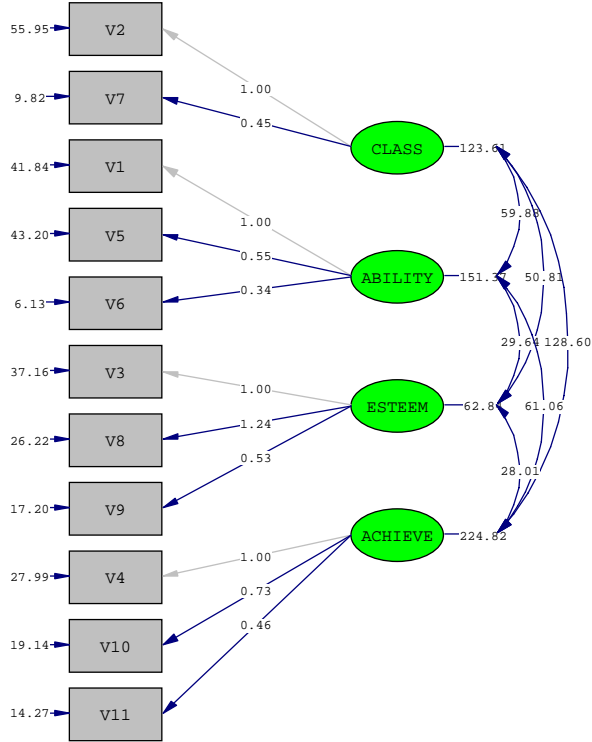
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TI boys group
DA NG=2 NI=11 NO=414 MA=CM
LA
V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
KM SY
1.00
.345 1.00
.287 .377 1.00
.252 .579 .114 1.00
.637 .335 .253 .255 1.00
.768 .339 .302 .278 .600 1.00
.254 .703 .337 .591 .312 .313 1.00
.166 .429 .724 .143 .264 .216 .364 1.00
.104 .411 .506 .142 .199 .163 .551 .630 1.00
.247 .601 .202 .880 .288 .309 .621 .290 .199 1.00
.208 .526 .127 .827 .253 .231 .676 .155 .234 .808 1.00
SD
13.9 13.4 10.0 15.9 9.4 4.9 5.9 11.1 5.9 11.8 7.9
SE
2 7 1 5 6 3 8 9 4 10 11
MO NX=11 NK=4 LX=FU, FI PH=SY,FR
LK
CLASS ABILITY ESTEEM ACHIEVE
FR LX 2 1 LX 4 2 LX 5 2 LX 7 3 LX 8 3 LX 10 4 LX 11 4
VA 1.0 LX 1 1 LX 3 2 LX 6 3 LX 9 4
PD
OU
TI girls group
DA NO=412
LA
V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
KM SY
1.00
.293 1.00
.278 .377 1.00
.109 .379 -.07 1.00
.656 .300 .211 .129 1.00
.825 .331 .311 .160 .677 1.00
.191 .659 .336 .378 .137 .263 1.00
.146 .400 .677 .042 .185 .211 .409 1.00
.168 .474 .554 .026 .140 .233 .626 .657 1.00
.123 .406 .026 .857 .186 .185 .408 .071 .049 1.00
.079 .362 .039 .822 .082 .124 .509 -.03 .080 .806 1.00
SD
14.5 11.0 9.5 16.3 8.9 5.0 4.9 10.7 5.3 11.1 7.1
SE
2 7 1 5 6 3 8 9 4 10 11
MO NX=11 NK=4 LX=FU, FI PH=SY,FR
LK
CLASS ABILITY ESTEEM ACHIEVE

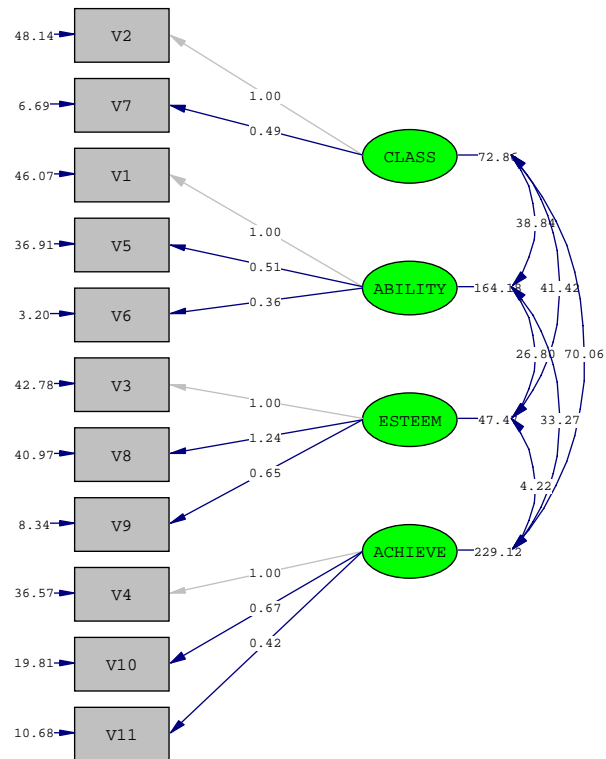
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FR LX 2 1 LX 4 2 LX 5 2 LX 7 3 LX 8 3 LX 10 4 LX 11 4
 VA 1.0 LX 1 1 LX 3 2 LX 6 3 LX 9 4
 PD
 OU

Boys :



Girls :



TI boys group

Parameter Specifications

LAMBDA-X						
	CLASS	ABILITY	ESTEEM	ACHIEVE		
	-----	-----	-----	-----		
V2	0	0	0	0		
V7	1	0	0	0		
V1	0	0	0	0		
V5	0	2	0	0		
V6	0	3	0	0		
V3	0	0	0	0		
V8	0	0	4	0		
V9	0	0	5	0		
V4	0	0	0	0		
V10	0	0	0	6		
V11	0	0	0	7		

PHI						
	CLASS	ABILITY	ESTEEM	ACHIEVE		
	-----	-----	-----	-----		
CLASS	8					
ABILITY	9	10				
ESTEEM	11	12	13			
ACHIEVE	14	15	16	17		

THETA-DELTA						
	CLASS	ABILITY	ESTEEM	ACHIEVE		
	-----	-----	-----	-----	-----	-----
V2		V7	V1	V5	V6	V3
18		19	20	21	22	23

THETA-DELTA						
	CLASS	ABILITY	ESTEEM	ACHIEVE		
	-----	-----	-----	-----	-----	-----
V8		V9	V4	V10	V11	
24		25	26	27	28	

TI girls group

Parameter Specifications

LAMBDA-X				
	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
V2	0	0	0	0
V7	29	0	0	0
V1	0	0	0	0
V5	0	30	0	0
V6	0	31	0	0
V3	0	0	0	0
V8	0	0	32	0
V9	0	0	33	0
V4	0	0	0	0
V10	0	0	0	34
V11	0	0	0	35

PHI				
	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
CLASS	36			
ABILITY	37	38		
ESTEEM	39	40	41	
ACHIEVE	42	43	44	45

THETA-DELTA					
V2	V7	V1	V5	V6	V3
-----	-----	-----	-----	-----	-----
46	47	48	49	50	51
THETA-DELTA					
V8	V9	V4	V10	V11	
-----	-----	-----	-----	-----	
52	53	54	55	56	

TI boys group

Number of Iterations = 11

LISREL Estimates (Maximum Likelihood)

LAMBDA-X				
	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
V2	1.00	- -	- -	- -
V7	0.45 (0.02) 18.81	- -	- -	- -
V1	- -	1.00	- -	- -
V5	- -	0.55 (0.03) 16.01	- -	- -
V6	- -	0.34 (0.02) 19.29	- -	- -
V3	- -	- -	1.00	- -
V8	- -	- -	1.24 (0.07) 17.03	- -
V9	- -	- -	0.53 (0.04) 14.59	- -
V4	- -	- -	- -	1.00
V10	- -	- -	- -	0.73 (0.02) 34.47
V11	- -	- -	- -	0.46 (0.02) 29.45
PHI				
	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
CLASS	123.61 (12.63) 9.79			

ABILITY	59.88 (8.71) 6.88	151.37 (14.30) 10.58		
ESTEEM	50.81 (6.23) 8.15	29.64 (5.85) 5.07	62.84 (6.99) 9.00	
ACHIEVE	128.60 (11.95) 10.76	61.06 (10.52) 5.81	28.01 (6.72) 4.17	224.82 (17.77) 12.65

THETA-DELTA	V2	V7	V1	V5	V6	V3
	-----	-----	-----	-----	-----	-----
	55.95 (5.80) 9.65	9.82 (1.11) 8.88	41.84 (6.39) 6.55	43.20 (3.50) 12.33	6.13 (0.80) 7.69	37.16 (3.71) 10.03

THETA-DELTA	V8	V9	V4	V10	V11
	-----	-----	-----	-----	-----
	26.22 (4.37) 6.01	17.20 (1.44) 11.97	27.99 (3.72) 7.53	19.14 (2.16) 8.84	14.27 (1.23) 11.61

Squared Multiple Correlations for X - Variables

V2	V7	V1	V5	V6	V3
-----	-----	-----	-----	-----	-----
0.69	0.72	0.78	0.51	0.74	0.63

Squared Multiple Correlations for X - Variables

V8	V9	V4	V10	V11
-----	-----	-----	-----	-----
0.79	0.51	0.89	0.86	0.77

Group Goodness of Fit Statistics

Contribution to Chi-Square = 293.34
 Percentage Contribution to Chi-Square = 47.20

Root Mean Square Residual (RMR) = 4.53
 Standardized RMR = 0.051
 Goodness of Fit Index (GFI) = 0.88

TI girls group

Number of Iterations = 11

LISREL Estimates (Maximum Likelihood)

LAMBDA-X	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
V2	1.00	- -	- -	- -
V7	0.49 (0.03)	- -	- -	- -

		16.32			
V1	- -	1.00	- -	- -	
V5	- -	0.51 (0.03) 17.67	- -	- -	
V6	- -	0.36 (0.02) 23.25	- -	- -	
V3	- -	- -	1.00	- -	
V8	- -	- -	1.24 (0.08) 14.76	- -	
V9	- -	- -	0.65 (0.04) 15.20	- -	
V4	- -	- -	- -	1.00	
V10	- -	- -	- -	0.67 (0.02) 30.75	
V11	- -	- -	- -	0.42 (0.01) 28.59	

	PHI			
	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
CLASS	72.86 (8.35) 8.73			
ABILITY	38.84 (6.82) 5.70	164.18 (15.14) 10.84		
ESTEEM	41.42 (4.80) 8.63	26.80 (5.33) 5.03	47.47 (5.94) 7.99	
ACHIEVE	70.06 (8.63) 8.12	33.27 (10.48) 3.17	4.22 (5.80) 0.73	229.12 (18.74) 12.22

THETA-DELTA

	V2	V7	V1	V5	V6	V3
	-----	-----	-----	-----	-----	-----
	48.14 (4.58) 10.51	6.69 (0.88) 7.65	46.07 (5.91) 7.80	36.91 (2.90) 12.75	3.20 (0.69) 4.62	42.78 (3.67) 11.67

THETA-DELTA				
V8	V9	V4	V10	V11
40.97	8.34	36.57	19.81	10.68
(4.14)	(0.98)	(4.56)	(2.20)	(1.00)
9.89	8.54	8.01	8.99	10.66

Squared Multiple Correlations for X - Variables					
V2	V7	V1	V5	V6	V3
0.60	0.72	0.78	0.53	0.87	0.53

Squared Multiple Correlations for X - Variables				
V8	V9	V4	V10	V11
0.64	0.70	0.86	0.84	0.79

Global Goodness of Fit Statistics

Degrees of Freedom = 76
 Minimum Fit Function Chi-Square = 621.51 (P = 0.0)
 Normal Theory Weighted Least Squares Chi-Square = 605.50 (P = 0.0)
 Estimated Non-centrality Parameter (NCP) = 529.50
 90 Percent Confidence Interval for NCP = (454.83 ; 611.64)

Minimum Fit Function Value = 0.75
 Population Discrepancy Function Value (F0) = 0.64
 90 Percent Confidence Interval for F0 = (0.55 ; 0.74)
 Root Mean Square Error of Approximation (RMSEA) = 0.13
 90 Percent Confidence Interval for RMSEA = (0.12 ; 0.14)
 P-Value for Test of Close Fit (RMSEA < 0.05) = 0.00

Expected Cross-Validation Index (ECVI) = 0.87
 90 Percent Confidence Interval for ECVI = (0.78 ; 0.97)
 ECVI for Saturated Model = 0.16
 ECVI for Independence Model = 9.44

Chi-Square for Independence Model with 110 Degrees of Freedom = 7755.49
 Independence AIC = 7799.49
 Model AIC = 717.50
 Saturated AIC = 264.00
 Independence CAIC = 7925.25
 Model CAIC = 1037.63
 Saturated CAIC = 1018.59

Normed Fit Index (NFI) = 0.92
 Non-Normed Fit Index (NNFI) = 0.90
 Parsimony Normed Fit Index (PNFI) = 0.64
 Comparative Fit Index (CFI) = 0.93
 Incremental Fit Index (IFI) = 0.93
 Relative Fit Index (RFI) = 0.88

Critical N (CN) = 143.64

Group Goodness of Fit Statistics

Contribution to Chi-Square = 328.17
 Percentage Contribution to Chi-Square = 52.80

Root Mean Square Residual (RMR) = 4.23
 Standardized RMR = 0.048
 Goodness of Fit Index (GFI) = 0.88

Boys and girls combined, fully constrained factor loadings:

```
TI boys group
DA NG=2 NI=11 NO=414 MA=CM
LA
V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
KM SY
1.00
.345 1.00
.287 .377 1.00
.252 .579 .114 1.00
.637 .335 .253 .255 1.00
.768 .339 .302 .278 .600 1.00
.254 .703 .337 .591 .312 .313 1.00
.166 .429 .724 .143 .264 .216 .364 1.00
.104 .411 .506 .142 .199 .163 .551 .630 1.00
.247 .601 .202 .880 .288 .309 .621 .290 .199 1.00
.208 .526 .127 .827 .253 .231 .676 .155 .234 .808 1.00
SD
13.9 13.4 10.0 15.9 9.4 4.9 5.9 11.1 5.9 11.8 7.9
SE
2 7 1 5 6 3 8 9 4 10 11
MO NX=11 NK=4 LX=FU, FI PH=SY,FR
LK
CLASS ABILITY ESTEEM ACHIEVE
FR LX 2 1 LX 4 2 LX 5 2 LX 7 3 LX 8 3 LX 10 4 LX 11 4
VA 1.0 LX 1 1 LX 3 2 LX 6 3 LX 9 4
PD
OU MI
TI girls group
DA NO=412
LA
V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
KM SY
1.00
.293 1.00
.278 .377 1.00
.109 .379 -.07 1.00
.656 .300 .211 .129 1.00
.825 .331 .311 .160 .677 1.00
.191 .659 .336 .378 .137 .263 1.00
.146 .400 .677 .042 .185 .211 .409 1.00
.168 .474 .554 .026 .140 .233 .626 .657 1.00
.123 .406 .026 .857 .186 .185 .408 .071 .049 1.00
.079 .362 .039 .822 .082 .124 .509 -.03 .080 .806 1.00
SD
14.5 11.0 9.5 16.3 8.9 5.0 4.9 10.7 5.3 11.1 7.1
SE
2 7 1 5 6 3 8 9 4 10 11
MO NX=11 NK=4 LX=FU, FI PH=SY,FR
LK
CLASS ABILITY ESTEEM ACHIEVE
FR LX 2 1 LX 4 2 LX 5 2 LX 7 3 LX 8 3 LX 10 4 LX 11 4
VA 1.0 LX 1 1 LX 3 2 LX 6 3 LX 9 4
EQ LX 1 2 1 LX 2 1
EQ LX 1 4 2 LX 4 2
EQ LX 1 5 2 LX 5 2
EQ LX 1 7 3 LX 7 3
EQ LX 1 8 3 LX 8 3
EQ LX 1 10 4 LX 10 4
EQ LX 1 11 4 LX 11 4
PD
OU MI
```

TI boys group

Parameter Specifications

LAMBDA-X EQUALS LAMBDA-X IN THE FOLLOWING GROUP

PHI						
	CLASS	ABILITY	ESTEEM	ACHIEVE		
	-----	-----	-----	-----		
CLASS	8					
ABILITY	9	10				
ESTEEM	11	12	13			
ACHIEVE	14	15	16	17		
THETA-DELTA						
	V2	V7	V1	V5	V6	V3
	-----	-----	-----	-----	-----	-----
	18	19	20	21	22	23
THETA-DELTA						
	V8	V9	V4	V10	V11	
	-----	-----	-----	-----	-----	
	24	25	26	27	28	

TI girls group

Parameter Specifications

LAMBDA-X						
	CLASS	ABILITY	ESTEEM	ACHIEVE		
	-----	-----	-----	-----		
V2	0	0	0	0		
V7	1	0	0	0		
V1	0	0	0	0		
V5	0	2	0	0		
V6	0	3	0	0		
V3	0	0	0	0		
V8	0	0	4	0		
V9	0	0	5	0		
V4	0	0	0	0		
V10	0	0	0	6		
V11	0	0	0	7		
PHI						
	CLASS	ABILITY	ESTEEM	ACHIEVE		
	-----	-----	-----	-----		
CLASS	29					
ABILITY	30	31				
ESTEEM	32	33	34			
ACHIEVE	35	36	37	38		
THETA-DELTA						
	V2	V7	V1	V5	V6	V3
	-----	-----	-----	-----	-----	-----
	39	40	41	42	43	44
THETA-DELTA						
	V8	V9	V4	V10	V11	
	-----	-----	-----	-----	-----	
	45	46	47	48	49	

TI boys group

Number of Iterations = 9

LISREL Estimates (Maximum Likelihood)

LAMBDA-X EQUALS LAMBDA-X IN THE FOLLOWING GROUP

PHI		CLASS	ABILITY	ESTEEM	ACHIEVE			
		-----	-----	-----	-----			
CLASS		119.82 (11.51) 10.41						
ABILITY		58.23 (8.37) 6.96	148.77 (12.82) 11.60					
ESTEEM		49.81 (5.84) 8.53	28.92 (5.63) 5.14	60.39 (6.00) 10.06				
ACHIEVE		128.98 (11.71) 11.01	61.87 (10.57) 5.86	27.81 (6.69) 4.16	234.46 (17.88) 13.11			
THETA-DELTA								
		V2	V7	V1	V5	V6	V3	
		-----	-----	-----	-----	-----	-----	
		57.15 (5.64) 10.14	9.58 (1.08) 8.83	43.50 (5.84) 7.46	44.10 (3.47) 12.72	5.79 (0.75) 7.74	37.95 (3.56) 10.67	
THETA-DELTA								
		V8	V9	V4	V10	V11		
		-----	-----	-----	-----	-----		
		28.37 (3.97) 7.14	16.55 (1.42) 11.68	26.39 (3.70) 7.14	19.74 (2.13) 9.27	14.53 (1.22) 11.90		

Squared Multiple Correlations for X - Variables

V2	V7	V1	V5	V6	V3
-----	-----	-----	-----	-----	-----
0.68	0.73	0.77	0.48	0.76	0.61

Squared Multiple Correlations for X - Variables

V8	V9	V4	V10	V11
-----	-----	-----	-----	-----
0.77	0.55	0.90	0.86	0.76

Group Goodness of Fit Statistics

Contribution to Chi-Square = 299.70
Percentage Contribution to Chi-Square = 47.18

Root Mean Square Residual (RMR) = 5.09
Standardized RMR = 0.054
Goodness of Fit Index (GFI) = 0.88

TI girls group

Number of Iterations = 9

LISREL Estimates (Maximum Likelihood)

LAMBDA-X				
	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
V2	1.00	- -	- -	- -
V7	0.46 (0.02) 24.83	- -	- -	- -
V1	- -	1.00	- -	- -
V5	- -	0.52 (0.02) 23.86	- -	- -
V6	- -	0.36 (0.01) 30.35	- -	- -
V3	- -	- -	1.00	- -
V8	- -	- -	1.24 (0.05) 22.97	- -
V9	- -	- -	0.58 (0.03) 21.50	- -
V4	- -	- -	- -	1.00
V10	- -	- -	- -	0.71 (0.02) 45.93
V11	- -	- -	- -	0.44 (0.01) 40.78
PHI				
	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
CLASS	77.04 (7.68) 10.03			
ABILITY	40.92 (6.96) 5.88	167.01 (14.28) 11.69		
ESTEEM	43.31 (4.67) 9.28	28.40 (5.52) 5.14	52.15 (5.32) 9.80	
ACHIEVE	70.22 (8.31) 8.45	32.26 (10.22) 3.16	4.12 (5.88) 0.70	214.39 (16.82) 12.74

THETA-DELTA					
V2	V7	V1	V5	V6	V3
46.06	7.18	44.19	36.38	3.53	39.86
(4.50)	(0.84)	(5.48)	(2.87)	(0.62)	(3.50)
10.24	8.55	8.06	12.65	5.70	11.38

THETA-DELTA				
V8	V9	V4	V10	V11
36.78	9.64	39.27	19.29	10.49
(3.95)	(0.95)	(4.37)	(2.17)	(1.00)
9.31	10.15	8.98	8.90	10.49

Squared Multiple Correlations for X - Variables					
V2	V7	V1	V5	V6	V3
0.63	0.70	0.79	0.56	0.86	0.57

Squared Multiple Correlations for X - Variables				
V8	V9	V4	V10	V11
0.69	0.64	0.85	0.85	0.80

Global Goodness of Fit Statistics

Degrees of Freedom = 83
 Minimum Fit Function Chi-Square = 635.28 (P = 0.0)
 Normal Theory Weighted Least Squares Chi-Square = 621.52 (P = 0.0)
 Estimated Non-centrality Parameter (NCP) = 538.52
 90 Percent Confidence Interval for NCP = (462.99 ; 621.52)

Minimum Fit Function Value = 0.77
 Population Discrepancy Function Value (F0) = 0.65
 90 Percent Confidence Interval for F0 = (0.56 ; 0.75)
 Root Mean Square Error of Approximation (RMSEA) = 0.13
 90 Percent Confidence Interval for RMSEA = (0.12 ; 0.13)
 P-Value for Test of Close Fit (RMSEA < 0.05) = 0.00

Expected Cross-Validation Index (ECVI) = 0.87
 90 Percent Confidence Interval for ECVI = (0.78 ; 0.97)
 ECVI for Saturated Model = 0.16
 ECVI for Independence Model = 9.44

Chi-Square for Independence Model with 110 Degrees of Freedom = 7755.49
 Independence AIC = 7799.49
 Model AIC = 719.52
 Saturated AIC = 264.00
 Independence CAIC = 7925.25
 Model CAIC = 999.63
 Saturated CAIC = 1018.59

Normed Fit Index (NFI) = 0.92
 Non-Normed Fit Index (NNFI) = 0.90
 Parsimony Normed Fit Index (PNFI) = 0.69
 Comparative Fit Index (CFI) = 0.93
 Incremental Fit Index (IFI) = 0.93
 Relative Fit Index (RFI) = 0.89

Critical N (CN) = 151.30

Group Goodness of Fit Statistics

Contribution to Chi-Square = 335.58
 Percentage Contribution to Chi-Square = 52.82

Root Mean Square Residual (RMR) = 4.38
 Standardized RMR = 0.050
 Goodness of Fit Index (GFI) = 0.88

To compare this model with the unconstrained one, we examine the difference of chi-squares and evaluate whether the fit of the constrained model is significantly worse than that of the unconstrained.

Chi-square of unconstrained model was 605.50, $df=76$; chi-square of constrained model is 621.52, $df=83$. Thus we get $621.52-605.50=16.02$, $df=83-76=7$ (that's 7 factor loadings that we do not estimate separately for boys in the constrained model). The p-value that corresponds to this chi-square is .025. Thus, if we pick $\alpha=.05$, we conclude that the constrained and unconstrained models differ significantly. Since the constrained model fits worse than unconstrained, we should try to pinpoint the source – i.e. which loadings actually differ. To do that, we can examine modification indices to assess which elements should be estimated separately for different groups:

TI boys group

Modification Indices and Expected Change

	Modification Indices for LAMBDA-X			
	CLASS	ABILITY	ESTEEM	ACHIEVE
V2	0.58	4.75	1.53	0.13
V7	0.58	4.75	1.53	0.13
V1	4.39	0.08	5.05	2.99
V5	7.96	1.65	7.98	3.81
V6	0.04	1.66	0.14	0.19
V3	0.70	13.47	0.51	0.90
V8	2.24	2.97	1.07	0.03
V9	7.28	3.53	3.31	0.67
V4	22.73	1.15	27.03	6.01
V10	7.94	2.97	27.25	1.38
V11	6.60	0.52	0.14	2.23

	Expected Change for LAMBDA-X			
	CLASS	ABILITY	ESTEEM	ACHIEVE
V2	0.06	0.10	0.10	-0.02
V7	-0.01	-0.05	-0.05	0.01
V1	-0.10	0.02	-0.14	-0.05
V5	0.10	0.03	0.14	0.05
V6	0.00	-0.02	0.01	0.00
V3	-0.03	0.12	0.06	-0.02
V8	-0.07	-0.06	0.04	0.00
V9	0.07	-0.04	-0.04	0.01
V4	-0.22	-0.03	-0.26	-0.10
V10	0.09	0.04	0.20	0.01
V11	0.06	-0.01	0.01	0.01

TI girls group

Modification Indices and Expected Change

Modification Indices for LAMBDA-X

	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
V2	0.58	7.71	0.56	0.44
V7	0.88	7.71	0.56	0.44
V1	4.83	0.08	3.33	3.62
V5	0.16	1.44	0.25	0.35
V6	5.58	1.50	4.27	2.05
V3	5.77	9.38	0.51	1.82
V8	7.13	4.71	1.12	0.00
V9	25.71	0.30	3.60	1.60
V4	1.44	0.02	3.41	6.01
V10	0.00	2.59	2.17	1.50
V11	1.78	3.59	0.16	2.35

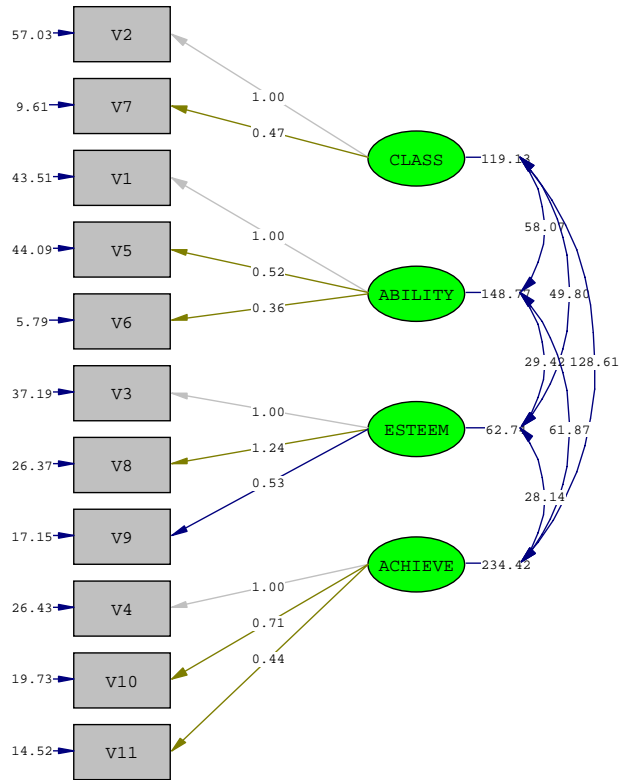
Expected Change for LAMBDA-X

	CLASS	ABILITY	ESTEEM	ACHIEVE
	-----	-----	-----	-----
V2	-0.06	0.10	-0.06	-0.02
V7	0.02	-0.05	0.03	0.01
V1	-0.12	-0.02	-0.12	-0.06
V5	-0.02	-0.02	-0.02	0.01
V6	0.04	0.01	0.04	0.01
V3	-0.13	0.09	-0.06	-0.03
V8	-0.15	-0.07	-0.05	0.00
V9	0.14	-0.01	0.04	0.02
V4	-0.06	0.00	-0.11	0.10
V10	0.00	0.04	0.06	-0.02
V11	0.03	-0.03	0.01	-0.01

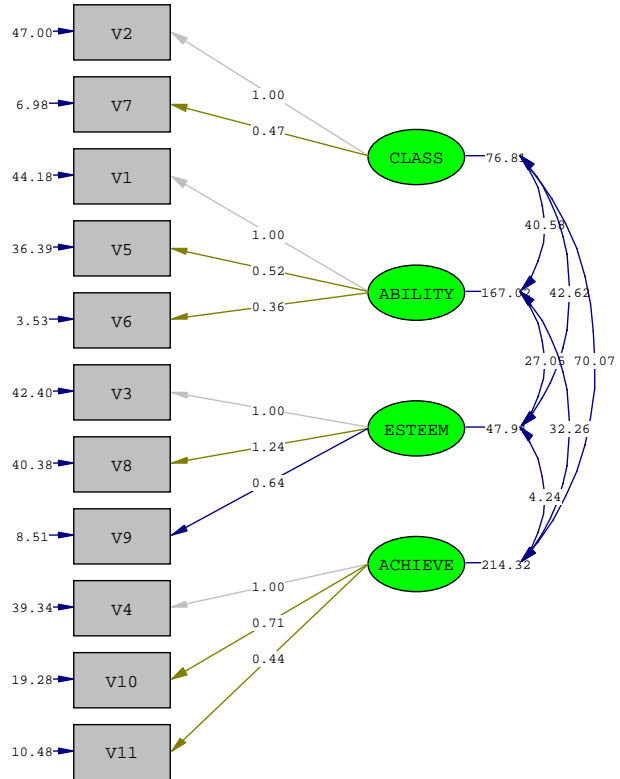
These indices would suggest that allowing the loadings for V9 on ESTEEM to vary would improve the fit of the constrained model. Importantly, if the measurement model varies by group, this means that the meaning of the latent variable differs across groups. If we are only interested in comparing measurement models across groups, that's fine, regardless of how different the measurement models turn out to be. But when we want to examine whether the structural model differs across groups, and yet we find that the measurement model differs across groups, then we are forced to make a decision how to define the latent variables for the purposes of comparing the structural models. That is, do we assume that the latent variable is conceptually the same across groups, even though the loadings are different? Or do we have to constrain the loadings to be the same for the purposes of comparability? This is known as exact replication versus conceptual replication (as the same kind of problem emerges if we try to measure the same latent variable over time). Alternatively, we could assume that some of the latent variables are indeed different across groups, and only compare those portions of the structural model that deal with similar latent variables.

For our example, we can free V9 coefficients from equality constraints, reestimate the model and use chi-square to compare that model to fully unconstrained:

Boys:



Girls:



Chi-Square=612.61, df=82, P-value=0.00000, RMSEA=0.125

With this one constraint lifted, the chi-square is much improved: 612.61. The difference between this model and unconstrained one is: $612.61 - 605.50 = 7.11$; $df=6$, and p-value associated with that is .31. But the loadings for V9 don't differ dramatically, so we will continue with our example by assuming invariance of the measurement model (that's what we would conclude if we used $\alpha=.01$).

2. Structural model.

Next, we can use constrained and unconstrained model to examine whether structural model is the same across groups. At this step, we usually made a decision regarding the measurement model. Suppose that we found measurement model to be equivalent. Then we use LX and LY matrices as invariant, and compare the model and estimates the elements of BE and GA matrices separately by group to the model that assumes them to be equal.

The algorithm of model modification is the same – we examine the unconstrained and constrained model, and if the constrained model is unacceptable, then we search for an acceptable partially constrained model. Again, you can either systematically go through the possibilities, one at a time, constraining or freeing the structural path coefficients, or use residuals and modification indices as the guidelines. Note, however, that if the goal is cross-validation, it doesn't really make sense to create such a partially constrained model. For our example, again, first, we would estimate separate models for boys and girls (I'll skip that step here for brevity), then the unconstrained model for the two groups together, and then the constrained model.

Unconstrained structural model for boys and girls:

```

TI boys group
DA NI=11 NG=2 NO=414 MA=CM
LA
V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
KM SY
1.00
.345 1.00
.287 .377 1.00
.252 .579 .114 1.00
.637 .335 .253 .255 1.00
.768 .339 .302 .278 .600 1.00
.254 .703 .337 .591 .312 .313 1.00
.166 .429 .724 .143 .264 .216 .364 1.00
.104 .411 .506 .142 .199 .163 .551 .630 1.00
.247 .601 .202 .880 .288 .309 .621 .290 .199 1.00
.208 .526 .127 .827 .253 .231 .676 .155 .234 .808 1.00
SD
13.9 13.4 10.0 15.9 9.4 4.9 5.9 11.1 5.9 11.8 7.9
SE
3 8 9 4 10 11 2 7 1 5 6

```

MO NX=5 NK=2 NY=6 NE=2 LX=FU, FI LY=FU,FI PH=SY,FR PS=DI,FR TD=DI,FR
TE=DI,FR BE=FU,FI GA=FU,FR

LK

CLASS ABILITY

LE

ESTEEM ACHIEVE

FR LX 2 1 LX 4 2 LX 5 2 LY 2 1 LY 3 1 LY 5 2 LY 6 2 BE 2 1

FI GA 1 2

VA 1.0 LX 1 1 LX 3 2 LY 1 1 LY 4 2

PD

OU

TI girls group

DA NO=412

LA

V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11

KM SY

1.00

.293 1.00

.278 .377 1.00

.109 .379 -.07 1.00

.656 .300 .211 .129 1.00

.825 .331 .311 .160 .677 1.00

.191 .659 .336 .378 .137 .263 1.00

.146 .400 .677 .042 .185 .211 .409 1.00

.168 .474 .554 .026 .140 .233 .626 .657 1.00

.123 .406 .026 .857 .186 .185 .408 .071 .049 1.00

.079 .362 .039 .822 .082 .124 .509 -.03 .080 .806 1.00

SD

14.5 11.0 9.5 16.3 8.9 5.0 4.9 10.7 5.3 11.1 7.1

SE

3 8 9 4 10 11 2 7 1 5 6

MO NX=5 NK=2 NY=6 NE=2 LX=IN LY=IN PH=SY, FR PS=DI,FR TD=DI,FR TE=DI,FR
BE=FU,FI GA=FU,FR

LK

CLASS ABILITY

LE

ESTEEM ACHIEVE

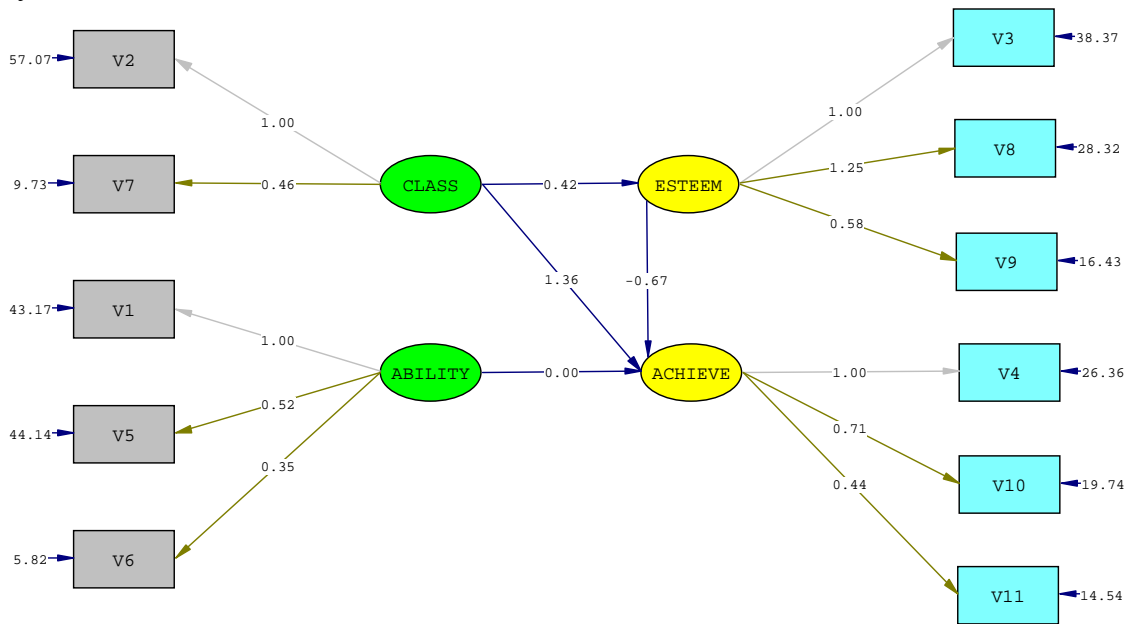
FR BE 2 1

FI GA 1 2

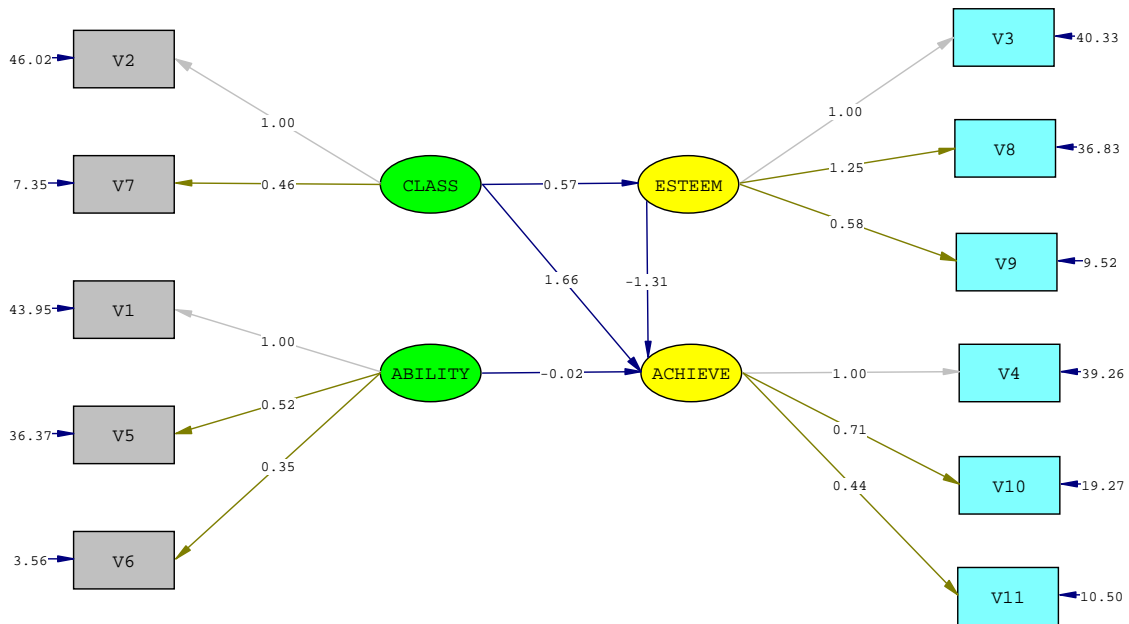
PD

OU

Boys:



Girls:



Chi-Square=626.77, df=85, P-value=0.00000, RMSEA=0.124

Constrained structural model for boys and girls:

TI boys group

DA NI=11 NG=2 NO=414 MA=CM

LA

V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11

KM SY

1.00

.345 1.00

.287 .377 1.00

.252 .579 .114 1.00

.637 .335 .253 .255 1.00

.768 .339 .302 .278 .600 1.00

.254 .703 .337 .591 .312 .313 1.00

.166 .429 .724 .143 .264 .216 .364 1.00

.104 .411 .506 .142 .199 .163 .551 .630 1.00

.247 .601 .202 .880 .288 .309 .621 .290 .199 1.00

.208 .526 .127 .827 .253 .231 .676 .155 .234 .808 1.00

SD

13.9 13.4 10.0 15.9 9.4 4.9 5.9 11.1 5.9 11.8 7.9

SE

3 8 9 4 10 11 2 7 1 5 6

MO NX=5 NK=2 NY=6 NE=2 LX=FU, FI LY=FU,FI PH=SY,FR PS=DI,FR TD=DI,FR
TE=DI,FR BE=FU,FI GA=FU,FR

LK

CLASS ABILITY

LE

ESTEEM ACHIEVE

FR LX 2 1 LX 4 2 LX 5 2 LY 2 1 LY 3 1 LY 5 2 LY 6 2 BE 2 1

FI GA 1 2

VA 1.0 LX 1 1 LX 3 2 LY 1 1 LY 4 2

PD

OU MI

TI girls group

DA NO=412

LA

V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11

KM SY

1.00

.293 1.00

.278 .377 1.00

.109 .379 -.07 1.00

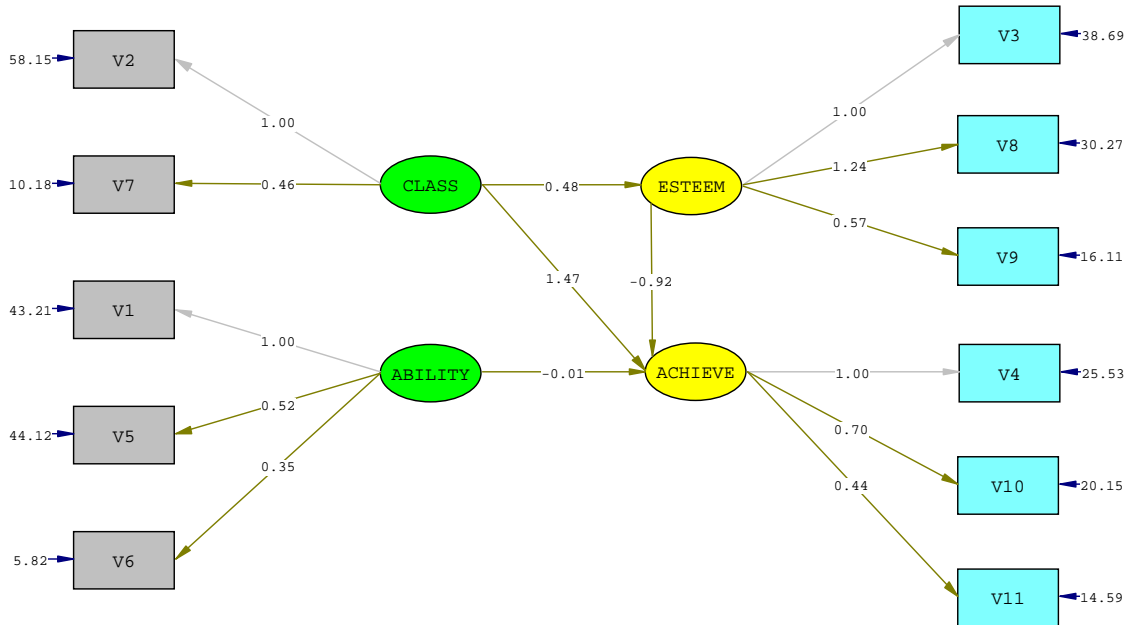
.656 .300 .211 .129 1.00

.825 .331 .311 .160 .677 1.00

.191 .659 .336 .378 .137 .263 1.00

.146 .400 .677 .042 .185 .211 .409 1.00

.168 .474 .554 .026 .140 .233 .626 .657 1.00
 .123 .406 .026 .857 .186 .185 .408 .071 .049 1.00
 .079 .362 .039 .822 .082 .124 .509 -.03 .080 .806 1.00
 SD
 14.5 11.0 9.5 16.3 8.9 5.0 4.9 10.7 5.3 11.1 7.1
 SE
 3 8 9 4 10 11 2 7 1 5 6
 MO NX=5 NK=2 NY=6 NE=2 LX=IN LY=IN PH=SY, FR PS=DI,FR TD=DI,FR TE=DI,FR
 BE=IN GA=IN
 LK
 CLASS ABILITY
 LE
 ESTEEM ACHIEVE
 PD
 OU MI



Chi-Square=636.76, df=89, P-value=0.00000, RMSEA=0.122

Chi-square test comparing this constrained model with the unconstrained: $636.76 - 626.77 = 9.99$, $df = 89 - 85 = 4$. The p-value that corresponds to this chi-square value is .04.

Check out the misspecification indices:

Boys:

Modification Indices for BETA

	ESTEEM	ACHIEVE
ESTEEM	4.53	0.41
ACHIEVE	7.09	0.13

Modification Indices for GAMMA

	CLASS -----	ABILITY -----
ESTEEM	4.53	0.01
ACHIEVE	2.51	0.61

Girls:

Modification Indices for BETA

	ESTEEM -----	ACHIEVE -----
ESTEEM	4.53	0.04
ACHIEVE	8.83	0.13

Modification Indices for GAMMA

	CLASS -----	ABILITY -----
ESTEEM	6.96	4.38
ACHIEVE	3.64	0.54

Looks like the effect of ESTEEM on ACHIEVE, and perhaps that of CLASS on ESTEEM, differs by group (Note: cutoffs for df=1 are 3.84 (.05) and 6.64 (.01)). Let's free up the effect of ESTEEM on ACHIEVE:

TI boys group

DA NI=11 NG=2 NO=414 MA=CM

LA

V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11

KM SY

1.00

.345 1.00

.287 .377 1.00

.252 .579 .114 1.00

.637 .335 .253 .255 1.00

.768 .339 .302 .278 .600 1.00

.254 .703 .337 .591 .312 .313 1.00

.166 .429 .724 .143 .264 .216 .364 1.00

.104 .411 .506 .142 .199 .163 .551 .630 1.00

.247 .601 .202 .880 .288 .309 .621 .290 .199 1.00

.208 .526 .127 .827 .253 .231 .676 .155 .234 .808 1.00

SD

13.9 13.4 10.0 15.9 9.4 4.9 5.9 11.1 5.9 11.8 7.9

SE

3 8 9 4 10 11 2 7 1 5 6

MO NX=5 NK=2 NY=6 NE=2 LX=FU, FI LY=FU, FI PH=SY, FR PS=DI, FR TD=DI, FR

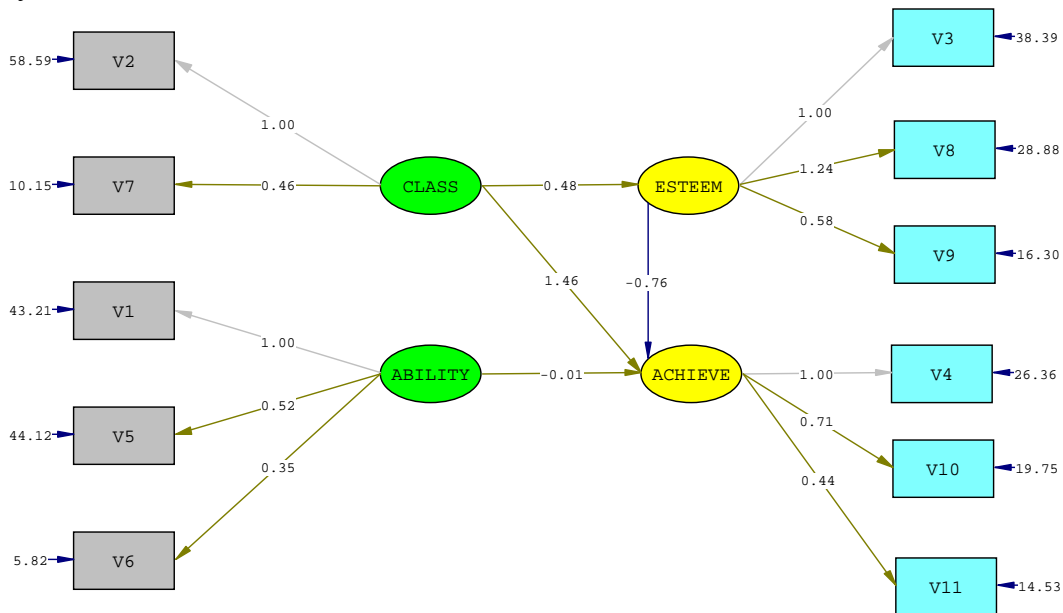
TE=DI, FR BE=FU, FI GA=FU, FR

LK

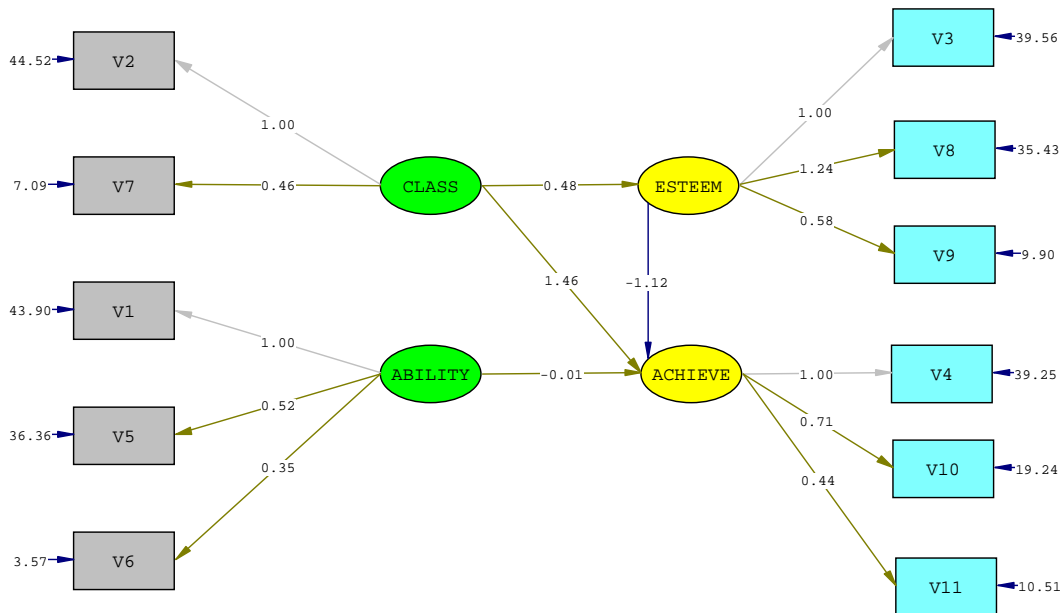
CLASS ABILITY

LE
 ESTEEM ACHIEVE
 FR LX 2 1 LX 4 2 LX 5 2 LY 2 1 LY 3 1 LY 5 2 LY 6 2 BE 2 1
 FI GA 1 2
 VA 1.0 LX 1 1 LX 3 2 LY 1 1 LY 4 2
 PD
 OU
 TI girls group
 DA NO=412
 LA
 V1 V2 V3 V4 V5 V6 V7 V8 V9 V10 V11
 KM SY
 1.00
 .293 1.00
 .278 .377 1.00
 .109 .379 -.07 1.00
 .656 .300 .211 .129 1.00
 .825 .331 .311 .160 .677 1.00
 .191 .659 .336 .378 .137 .263 1.00
 .146 .400 .677 .042 .185 .211 .409 1.00
 .168 .474 .554 .026 .140 .233 .626 .657 1.00
 .123 .406 .026 .857 .186 .185 .408 .071 .049 1.00
 .079 .362 .039 .822 .082 .124 .509 -.03 .080 .806 1.00
 SD
 14.5 11.0 9.5 16.3 8.9 5.0 4.9 10.7 5.3 11.1 7.1
 SE
 3 8 9 4 10 11 2 7 1 5 6
 MO NX=5 NK=2 NY=6 NE=2 LX=IN LY=IN PH=SY, FR PS=DI,FR TD=DI,FR TE=DI,FR
 BE=FU, FI GA=IN
 LK
 CLASS ABILITY
 LE
 ESTEEM ACHIEVE
 FR BE 2 1
 PD
 OU

Boys:



Girls:



Chi-Square=629.77, df=88, P-value=0.00000, RMSEA=0.122

The difference in chi-square between this partially constrained model and the unconstrained model is rather minor: $629.77 - 626.77 = 3.00$, and $df = 88 - 85 = 3$. P-value for that is .39. So this model is not significantly different from the unconstrained.

Example of multigroup analysis:

Rick Kosterman, Kevin P. Haggerty, Richard Spoth, and Cleve Redmond. 2004. "Unique Influence of Mothers and Fathers on Their Children's Antisocial Behavior." *Journal of Marriage and Family*, 66: 762-778.