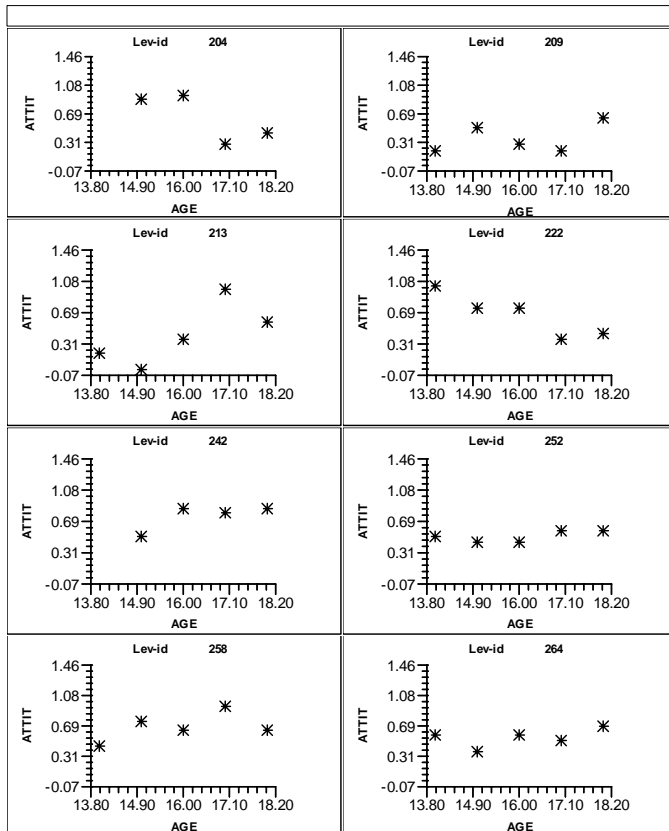


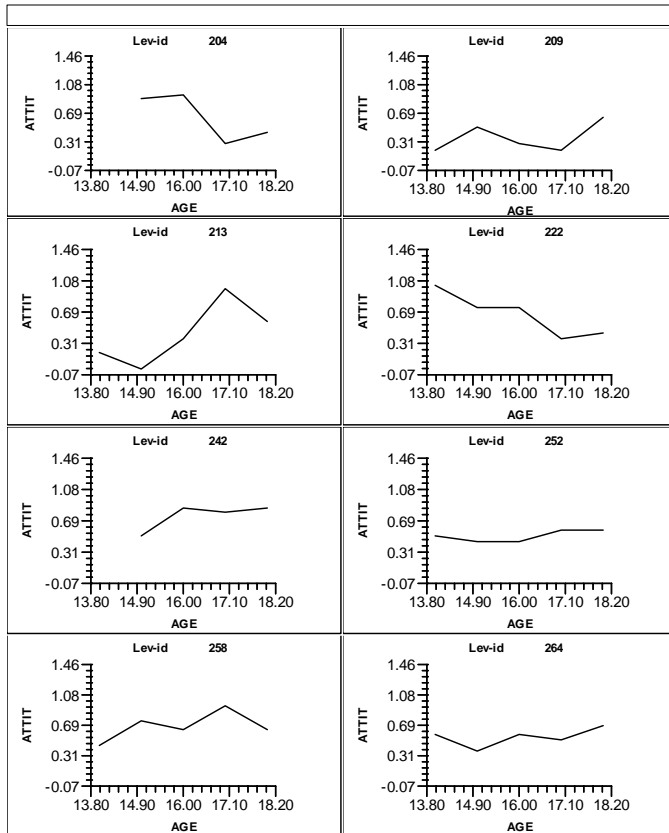
**October 18, 2007**  
**SC705: Advanced Statistics**  
**Instructor: Natasha Sarkisian**  
**Class notes: Growth Curve Models**

So far we've worked with one type of hierarchical data – students nested within schools. HLM can also be used to model longitudinal data where multiple observations over time are nested within one person.

We will use NYS2.MDM from Chapter 9 folder. This file contains data for a cohort of adolescents in the National Youth Survey, ages 14 to 18. The dependent variable *ATTIT* is a 9-item scale assessing attitudes favorable to deviant behavior (property damage, drug and alcohol use, stealing, etc.). The level-1 independent variables include: *EXPO* measuring exposure to deviant peers (students were asked how many of their friends engaged in the 9 deviant behaviors), *AGE* (age in years), *AGES* (age in years squared), *AGE14* (age minus 14), *AGE16* (age minus 16), *AGE145* (age minus 14.5), and the three corresponding squared variables. Level 2 include person-level variables: *FEMALE*, *MINORITY*, *INCOME*, and an interaction term for *MINFEM*.

What we will study is how attitudes change over time, and what shapes that change. First, let's examine individual trajectories.





Now let's try to model these trajectories. First, we will assume that we can model them using a linear model. Therefore, we'll estimate an unconditional linear growth model:

Level-1 Model

$$Y = B_0 + B_1 \cdot (\text{AGE16}) + R$$

Level-2 Model

$$B_0 = G_{00} + U_0$$

$$B_1 = G_{10} + U_1$$

Sigma\_squared = 0.02873

Tau

INTRCPT1,B0	0.04572	-0.00093
AGE16,B1	-0.00093	0.00313

Tau (as correlations)

INTRCPT1,B0	1.000	-0.078
AGE16,B1	-0.078	1.000

```

-----
Random level-1 coefficient   Reliability estimate
-----
INTRCPT1, B0                0.837
AGE16, B1                   0.453
-----

```

The outcome variable is ATTIT

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, B0					
INTRCPT2, G00	0.493325	0.014864	33.189	240	0.000
For AGE16 slope, B1					
INTRCPT2, G10	0.032357	0.005350	6.048	240	0.000

The outcome variable is ATTIT

Final estimation of fixed effects  
(with robust standard errors)

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, B0					
INTRCPT2, G00	0.493325	0.014833	33.259	240	0.000
For AGE16 slope, B1					
INTRCPT2, G10	0.032357	0.005338	6.061	240	0.000

Final estimation of variance components:

Random Effect	Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1, U0	0.21383	0.04572	235	1754.38522	0.000
AGE16 slope, U1	0.05595	0.00313	235	446.20764	0.000
level-1, R	0.16949	0.02873			

Statistics for current covariance components model

Deviance = -99.676230  
Number of estimated parameters = 4

The mean growth trajectory is:

$$\text{Attitude} = .493 + .032 * \text{Age16}$$

Now let's estimate an unconditional quadratic growth model and compare the fit:

Level-1 Model

$$Y = B0 + B1 * (\text{AGE16}) + B2 * (\text{AGE16S}) + R$$

Level-2 Model

$$B0 = G00 + U0$$

$$B1 = G10 + U1$$

$$B2 = G20 + U2$$

Sigma\_squared = 0.02291

Tau

INTRCPT1, B0	0.05825	-0.00033	-0.00416
AGE16, B1	-0.00033	0.00369	-0.00033



Our quadratic model does have smaller deviance value, but let's test the quadratic model against the linear model:

```
Variance-Covariance components test
-----
Chi-square statistic      =      29.93990
Number of degrees of freedom =      3
P-value                  =      0.000
```

We conclude that quadratic model is a better fit, and proceed to estimating conditional models using person-level (time-invariant) predictors at first.

The model specified for the fixed effects was:

```
-----
Level-1                               Level-2
Coefficients                           Predictors
-----
          INTRCPT1, B0                 INTRCPT2, G00
                                         FEMALE, G01
                                         MINORITY, G02
$          AGE16 slope, B1             INTRCPT2, G10
                                         FEMALE, G11
                                         MINORITY, G12
$          AGE16S slope, B2           INTRCPT2, G20
                                         FEMALE, G21
                                         MINORITY, G22
$                                         INTRCPT2, G23
                                         INCOME, G03
                                         INCOME, G13
                                         INCOME, G23
```

'\$' - This level-2 predictor has been centered around its grand mean.

Level-1 Model

$$Y = B0 + B1*(AGE16) + B2*(AGE16S) + R$$

Level-2 Model

$$B0 = G00 + G01*(FEMALE) + G02*(MINORITY) + G03*(INCOME) + U0$$

$$B1 = G10 + G11*(FEMALE) + G12*(MINORITY) + G13*(INCOME) + U1$$

$$B2 = G20 + G21*(FEMALE) + G22*(MINORITY) + G23*(INCOME) + U2$$

Sigma\_squared = 0.02291

```
Tau
INTRCPT1,B0    0.05662    -0.00042    -0.00391
AGE16,B1      -0.00042     0.00364    -0.00025
AGE16S,B2     -0.00391    -0.00025     0.00112
```

Tau (as correlations)

```
INTRCPT1,B0  1.000 -0.029 -0.492
AGE16,B1    -0.029  1.000 -0.122
AGE16S,B2   -0.492 -0.122  1.000
```

Random level-1 coefficient      Reliability estimate

```
-----
INTRCPT1, B0                               0.818
AGE16, B1                                  0.527
AGE16S, B2                                  0.346
-----
```

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, B0					
INTRCPT2, G00	0.562491	0.025856	21.754	237	0.000
FEMALE, G01	-0.100283	0.034929	-2.871	237	0.005
MINORITY, G02	-0.019852	0.044100	-0.450	237	0.653
INCOME, G03	0.003602	0.007755	0.464	237	0.642
For AGE16 slope, B1					
INTRCPT2, G10	0.039149	0.008110	4.827	237	0.000
FEMALE, G11	-0.003239	0.010823	-0.299	237	0.765
MINORITY, G12	-0.028441	0.013824	-2.057	237	0.040
INCOME, G13	-0.003963	0.002373	-1.670	237	0.096
For AGE16S slope, B2					
INTRCPT2, G20	-0.019852	0.005501	-3.609	237	0.001
FEMALE, G21	0.014754	0.007364	2.003	237	0.046
MINORITY, G22	0.012461	0.009468	1.316	237	0.190
INCOME, G23	0.002798	0.001620	1.727	237	0.085

Final estimation of fixed effects  
(with robust standard errors)

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, B0					
INTRCPT2, G00	0.562491	0.029658	18.966	237	0.000
FEMALE, G01	-0.100283	0.034379	-2.917	237	0.004
MINORITY, G02	-0.019852	0.039082	-0.508	237	0.611
INCOME, G03	0.003602	0.006930	0.520	237	0.603
For AGE16 slope, B1					
INTRCPT2, G10	0.039149	0.007686	5.094	237	0.000
FEMALE, G11	-0.003239	0.010304	-0.314	237	0.753
MINORITY, G12	-0.028441	0.014088	-2.019	237	0.044
INCOME, G13	-0.003963	0.002075	-1.910	237	0.057
For AGE16S slope, B2					
INTRCPT2, G20	-0.019852	0.006129	-3.239	237	0.002
FEMALE, G21	0.014754	0.007121	2.072	237	0.039
MINORITY, G22	0.012461	0.009555	1.304	237	0.194
INCOME, G23	0.002798	0.001383	2.023	237	0.044

Final estimation of variance components:

Random Effect	Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1, U0	0.23795	0.05662	219	1196.00045	0.000
AGE16 slope, U1	0.06030	0.00364	219	495.23926	0.000
AGE16S slope, U2	0.03342	0.00112	219	336.68827	0.000
level-1, R	0.15135	0.02291			

Finally, let's estimate a quadratic growth model with a time-varying covariate (EXPO). Here, we will use EXPO grand-centered. If we wanted to take this analysis one step further, we could have created a mean exposure variable on person level (level 2) and then used EXPO group centered on level 1 and mean of EXPO on level 2.

	Level-1 Coefficients	Level-2 Predictors
	INTRCPT1, B0	INTRCPT2, G00 FEMALE, G01 MINORITY, G02 INCOME, G03
\$		
%	EXPO slope, B1	INTRCPT2, G10 FEMALE, G11 MINORITY, G12 INCOME, G13
\$		
	AGE16 slope, B2	INTRCPT2, G20 FEMALE, G21 MINORITY, G22 INCOME, G23
\$		
	AGE16S slope, B3	INTRCPT2, G30 FEMALE, G31 MINORITY, G32 INCOME, G33
\$		

'%' - This level-1 predictor has been centered around its grand mean.  
'\$' - This level-2 predictor has been centered around its grand mean.

#### Level-1 Model

$$Y = B0 + B1*(EXPO) + B2*(AGE16) + B3*(AGE16S) + R$$

#### Level-2 Model

$$B0 = G00 + G01*(FEMALE) + G02*(MINORITY) + G03*(INCOME) + U0$$

$$B1 = G10 + G11*(FEMALE) + G12*(MINORITY) + G13*(INCOME) + U1$$

$$B2 = G20 + G21*(FEMALE) + G22*(MINORITY) + G23*(INCOME) + U2$$

$$B3 = G30 + G31*(FEMALE) + G32*(MINORITY) + G33*(INCOME) + U3$$

$$\text{Sigma\_squared} = 0.02030$$

#### Tau

INTRCPT1,B0	0.02273	-0.00288	0.00068	-0.00147
EXPO,B1	-0.00288	0.03327	-0.00273	0.00185
AGE16,B2	0.00068	-0.00273	0.00276	-0.00034
AGE16S,B3	-0.00147	0.00185	-0.00034	0.00058

#### Tau (as correlations)

INTRCPT1,B0	1.000	-0.105	0.086	-0.405
EXPO,B1	-0.105	1.000	-0.285	0.420
AGE16,B2	0.086	-0.285	1.000	-0.270
AGE16S,B3	-0.405	0.420	-0.270	1.000

Random level-1 coefficient	Reliability estimate
INTRCPT1, B0	0.342
EXPO, B1	0.062
AGE16, B2	0.330
AGE16S, B3	0.166

#### Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value

For	INTRCPT1, B0					
	INTRCPT2, G00	0.536548	0.018864	28.443	237	0.000
	FEMALE, G01	-0.087195	0.025319	-3.444	237	0.001
	MINORITY, G02	-0.003917	0.032033	-0.122	237	0.903
	INCOME, G03	0.006434	0.005601	1.149	237	0.252
For	EXPO slope, B1					
	INTRCPT2, G10	0.551921	0.041454	13.314	237	0.000
	FEMALE, G11	-0.048549	0.058298	-0.833	237	0.406
	MINORITY, G12	-0.404139	0.071710	-5.636	237	0.000
	INCOME, G13	-0.042315	0.013089	-3.233	237	0.002
For	AGE16 slope, B2					
	INTRCPT2, G20	0.018852	0.007483	2.519	237	0.013
	FEMALE, G21	0.008663	0.009922	0.873	237	0.384
	MINORITY, G22	-0.008015	0.012682	-0.632	237	0.528
	INCOME, G23	-0.001653	0.002179	-0.759	237	0.449
For	AGE16S slope, B3					
	INTRCPT2, G30	-0.011305	0.004845	-2.333	237	0.021
	FEMALE, G31	0.014522	0.006476	2.242	237	0.026
	MINORITY, G32	0.003959	0.008345	0.474	237	0.635
	INCOME, G33	0.002238	0.001416	1.580	237	0.115

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Final estimation of fixed effects  
(with robust standard errors)

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value	
For	INTRCPT1, B0					
	INTRCPT2, G00	0.536548	0.019993	26.837	237	0.000
	FEMALE, G01	-0.087195	0.024535	-3.554	237	0.001
	MINORITY, G02	-0.003917	0.032658	-0.120	237	0.905
	INCOME, G03	0.006434	0.005623	1.144	237	0.254
For	EXPO slope, B1					
	INTRCPT2, G10	0.551921	0.038407	14.370	237	0.000
	FEMALE, G11	-0.048549	0.057455	-0.845	237	0.399
	MINORITY, G12	-0.404139	0.072271	-5.592	237	0.000
	INCOME, G13	-0.042315	0.014359	-2.947	237	0.004
For	AGE16 slope, B2					
	INTRCPT2, G20	0.018852	0.007315	2.577	237	0.011
	FEMALE, G21	0.008663	0.009423	0.919	237	0.359
	MINORITY, G22	-0.008015	0.013110	-0.611	237	0.541
	INCOME, G23	-0.001653	0.002002	-0.826	237	0.410
For	AGE16S slope, B3					
	INTRCPT2, G30	-0.011305	0.004920	-2.298	237	0.022
	FEMALE, G31	0.014522	0.006166	2.355	237	0.019
	MINORITY, G32	0.003959	0.008755	0.452	237	0.651
	INCOME, G33	0.002238	0.001269	1.763	237	0.079

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Final estimation of variance components:

Random Effect	Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1, U0	0.15078	0.02273	197	402.26089	0.000
EXPO slope, U1	0.18240	0.03327	197	244.37700	0.012
AGE16 slope, U2	0.05252	0.00276	197	307.13303	0.000
AGE16S slope, U3	0.02415	0.00058	197	250.50418	0.006
level-1, R	0.14248	0.02030			