

# Longitudinal Data Analysis

## Instructor: Natasha Sarkisian

### Panel Data Analysis: Mixed Effects Models

So far, when analyzing panel data, we only allowed for the intercepts to vary across units (by having fixed effects or random effects for countries or individuals). A whole other class of models, mixed effects models, also known as multilevel models, hierarchical linear models, or growth curve models, allows for the coefficients themselves to vary across units. That is, we assume that the effects of time-varying variables, and time itself, are not the same across units. We will look at average effect of such variables, the extent to which there is variation around that average, and at level 2 (time-invariant) predictors that may explain that variation (so-called cross-level interactions). But first let's reexamine the equation for random effects model:

$$Y_{ij} = \alpha + X\beta + u_i + e_{ij}$$

We can also rewrite it as:

$$\text{Level 1 model is: } Y_{ij} = \alpha + X\beta + e_{ij}$$

$$\text{Level 2 model is: } \alpha = \pi_0 + u_i$$

Thus, we expressed a random effects model as a two-level model where we can explicitly see that the intercept for each unit equals to grand mean plus unit-specific residual. If our model also contains some time-invariant predictors, we can also write:

$$\text{Level 1 model is: } Y_{ij} = \alpha + X\beta + e_{ij}$$

$$\text{Level 2 model is: } \alpha = \pi_0 + X_i\beta_i + u_i$$

Moving beyond random effects models to mixed models, we can write a similar equation for each of level 1 regression coefficients:

$$\text{Level 1 model is: } Y_{ij} = \alpha + X\beta + e_{ij}$$

$$\text{Level 2 model is: } \alpha = \pi_0 + X_i\beta_i + u_{0i},$$

$$\beta_1 = \pi_1 + X_i\beta_i + u_{1i},$$

...

We will use an example that examines how attitudes toward deviant behavior change over time for teenagers, and what shapes that change. We will use a file called `nys.dta`. This file contains data for a cohort of adolescents in the National Youth Survey, ages 14 to 18. The dependent variable `attit` is a 9-item scale assessing attitudes favorable to deviant behavior (property damage, drug and alcohol use, stealing, etc.). The level-1 independent variables include: `expo` measuring exposure to deviant peers (students were asked how many of their friends engaged in the 9 deviant behaviors) and `age` (age in years). Level 2 include person-level variables: `female`, `minority`, and `income`.

```
. use http://www.sarkisian.net/sc706/nys.dta
```

```
. reshape long attit expo, i(id) j(age)
(note: j = 14 15 16 17 18)
```

```
Data                                wide  ->  long
```

---

```

Number of obs.          241  ->  1205
Number of variables     14  ->   7
j variable (5 values)   ->  age
xij variables:
    attit14 attit15 ... attit18  ->  attit
    expo14  expo15 ... expo18    ->  expo

```

```
-----
```

```
. egen miss=rowmiss( attit expo)
```

```
. tab miss
```

miss	Freq.	Percent	Cum.
0	1,066	88.46	88.46
2	139	11.54	100.00
Total	1,205	100.00	

```
. drop if miss==2
(139 observations deleted)
```

```
. xtset id age, yearly
      panel variable:  id (unbalanced)
      time variable:  age, 14 to 18, but with gaps
                   delta: 1 year
```

Remember: Data are considered strongly balanced if all the time points are the same and all cases are observed at all time points. Data are considered balanced if the cases have the same number of time values but these are not exactly the same time points. Data are unbalanced if cases are observed at different numbers of time points.

Focusing just on age, we could estimate a random effects model using both xtreg and xtmixed:

```
. xtreg attit age, re
```

```

Random-effects GLS regression              Number of obs   =       1066
Group variable: id                        Number of groups =        241

R-sq:  within = 0.0674                    Obs per group:  min =         1
        between = 0.0000                    avg =         4.4
        overall = 0.0207                    max =         5

```

```

Random effects u_i ~ Gaussian              Wald chi2(1)    =       58.31
corr(u_i, X) = 0 (assumed)                 Prob > chi2     =       0.0000

```

```
-----
```

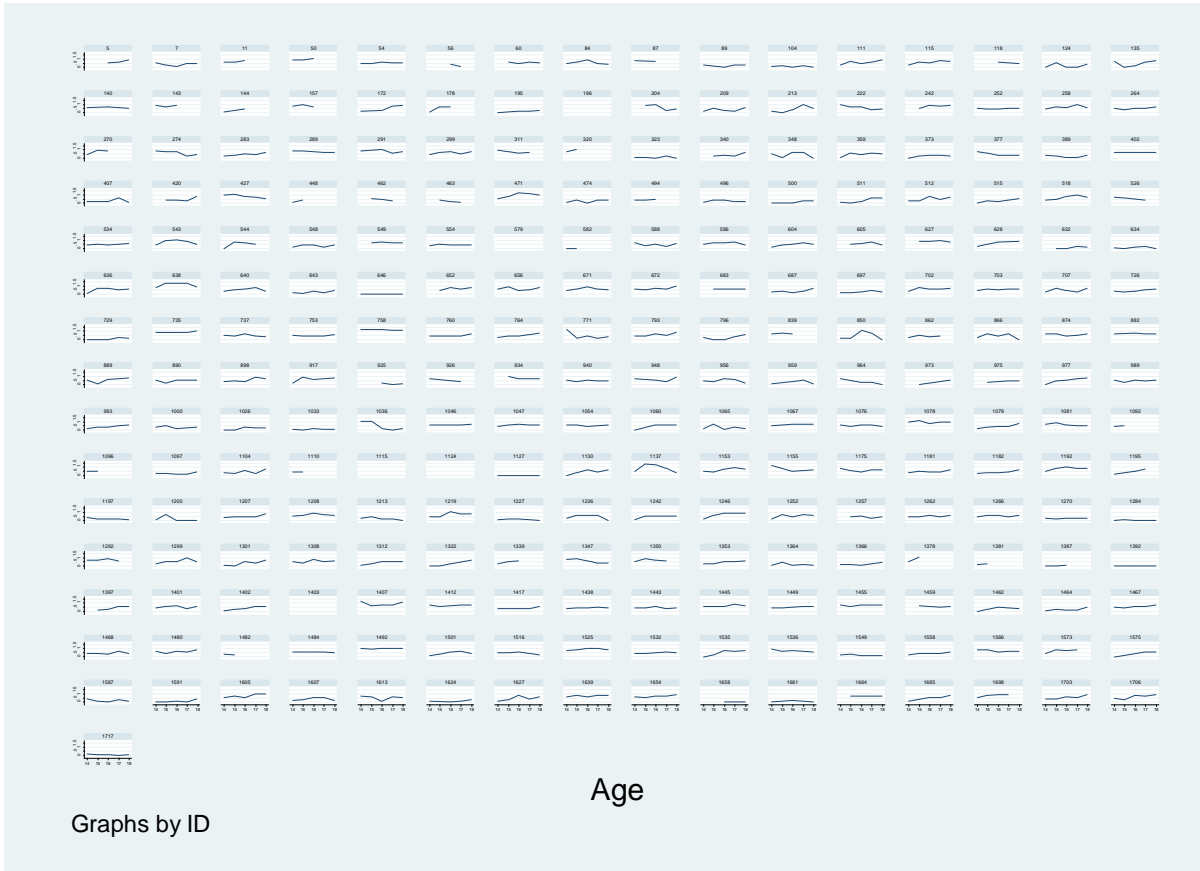
attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age	.0324074	.0042441	7.64	0.000	.0240892	.0407256
_cons	-.0258944	.0692441	-0.37	0.708	-.1616103	.1098215
sigma_u	.21445769					
sigma_e	.18975623					
rho	.5608825	(fraction of variance due to u_i)				

```
-----
```

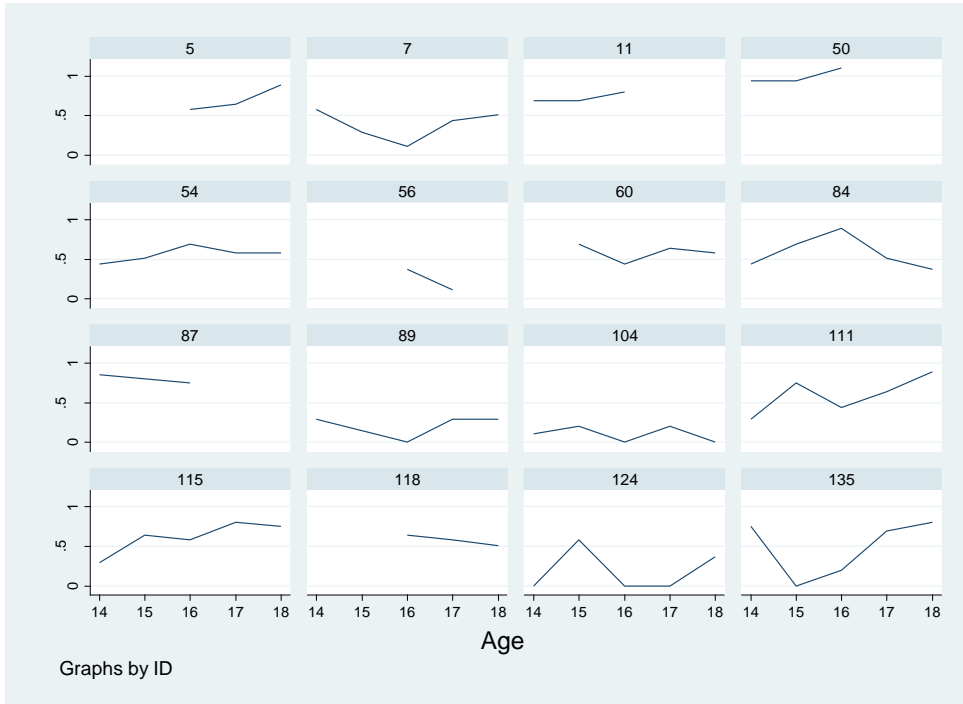
```
. xtmixed attit age || id:
```

```
Performing EM optimization:
```

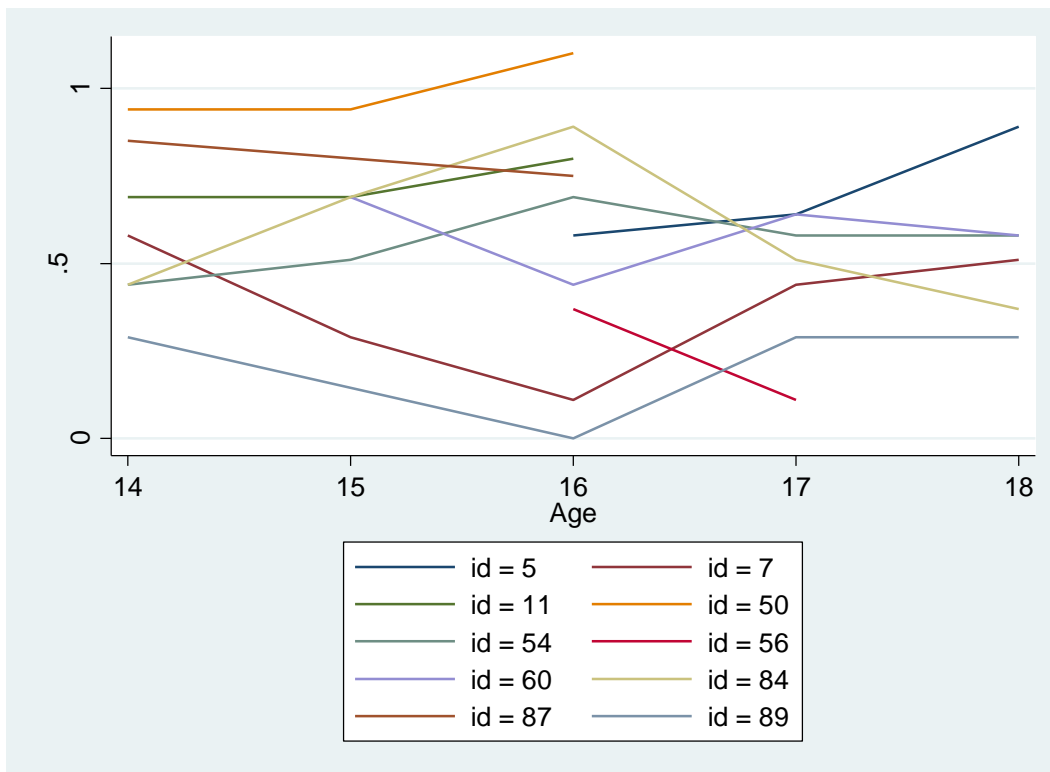




```
. xtline attit if id<100
```



```
. xtline attit if id<100, overlay
```



Very often, in this type of analysis, we are interested in understanding why and how the trajectory over time varies across units (that is why these models are also called growth curve

models), so we want to explore that variation – that requires estimating a mixed effects model; random effects model cannot assess variation in the slope of age.

```
. xtmixed attit age || id: age, cov(unstructured)
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0: log restricted-likelihood = 30.601124
Iteration 1: log restricted-likelihood = 45.872252
Iteration 2: log restricted-likelihood = 48.997375
Iteration 3: log restricted-likelihood = 49.825208
Iteration 4: log restricted-likelihood = 49.83764
Iteration 5: log restricted-likelihood = 49.838114
Iteration 6: log restricted-likelihood = 49.838114
```

Computing standard errors:

```
Mixed-effects REML regression          Number of obs      =      1066
Group variable: id                    Number of groups   =       241

Obs per group: min =          1
                  avg =         4.4
                  max =          5

Wald chi2(1) =          36.57
Prob > chi2   =          0.0000

Log restricted-likelihood = 49.838114
```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age	.0323571	.0053505	6.05	0.000	.0218702	.0428439
_cons	-.024388	.0872417	-0.28	0.780	-.1953785	.1466025

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]	
id: Unstructured				
sd(age)	.0559502	.0057241	.0457845	.0683731
sd(_cons)	.9364748	.0914994	.7732651	1.134132
corr(age, _cons)	-.9737465	.0059243	-.9831493	-.9592048
sd(Residual)	.1694919	.0048755	.1602004	.1793222

```
LR test vs. linear regression:      chi2(3) = 439.72   Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

Note that we specified covariance option – that is because we want to allow random effects to correlate with each other; if we do not, that would be too restrictive since usually random effects for intercepts and slopes are correlated. So we have two random effects now:

$$\begin{pmatrix} u_{0i} \\ u_{1i} \end{pmatrix} \sim N \left( 0, \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix} \right)$$

Our tau matrix now contains the variance in the level-1 intercepts ( $\tau_{00}$ ), the variance in level-1 slopes ( $\tau_{11}$ ), as well as the covariance between level-1 intercepts and slopes ( $\tau_{01} = \tau_{10}$ ). (This

covariance is presented as a correlation in our output.) Note that covariance value indicates how much intercepts and slopes covary: in our example, there is a negative correlation between intercepts and slopes. That is, the higher the intercept, the smaller the slope (i.e. if the starting point in terms of deviant attitudes is higher, then the slope is less steep). We can see this as a variance-covariance matrix:

```
. estat recov

Random-effects covariance matrix for level id
-----+-----
          |          age          _cons
-----+-----
    age |   .0031015
   _cons |  -.0505552   .8692899
```

So far we assumed that the time trend is linear but the graph above shows that for many people it is not. Let's estimate a model with a quadratic trend.

```
. tab age

      Age |      Freq.      Percent      Cum.
-----+-----
      14 |         241         20.00         20.00
      15 |         241         20.00         40.00
      16 |         241         20.00         60.00
      17 |         241         20.00         80.00
      18 |         241         20.00        100.00
-----+-----
    Total |       1,205        100.00
```

```
. gen age16=age-16
. gen age16sq=age16^2
```

Note that the intercept will now correspond to value at age 16 rather than at the start of the study.

```
. xtmixed attit age16 age16sq || id: age16 age16sq, cov(unstructured)

Performing EM optimization:

Performing gradient-based optimization:

Iteration 0:  log restricted-likelihood = 63.779001
Iteration 1:  log restricted-likelihood = 63.88902
Iteration 2:  log restricted-likelihood = 63.889125
Iteration 3:  log restricted-likelihood = 63.889125

Computing standard errors:

Mixed-effects REML regression
Group variable: id

Number of obs      =      1066
Number of groups   =       241

Obs per group: min =         1
                  avg =         4.4
                  max =         5

Log restricted-likelihood = 63.889125
Wald chi2(2)          =      41.34
Prob > chi2           =      0.0000
```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age16	.0314627	.0053327	5.90	0.000	.0210107	.0419146
age16sq	-.0106962	.0036517	-2.93	0.003	-.0178533	-.003539
_cons	.5140183	.0173066	29.70	0.000	.4800979	.5479387

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]	
id: Unstructured				
sd(age16)	.060747	.0052145	.0513402	.0718773
sd(age16sq)	.034373	.0044432	.0266801	.0442841
sd(_cons)	.2413507	.0137404	.2158682	.2698413
corr(age16,age16sq)	-.1603304	.1389152	-.4146204	.1171857
corr(age16,_cons)	-.0223911	.0975507	-.2104925	.1673091
corr(age16sq,_cons)	-.5016773	.086103	-.6510172	-.3149473
sd(Residual)	.1513583	.005323	.1412768	.1621592

LR test vs. linear regression:           chi2(6) =   471.17   Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

How does the average trajectory look like based on this model?

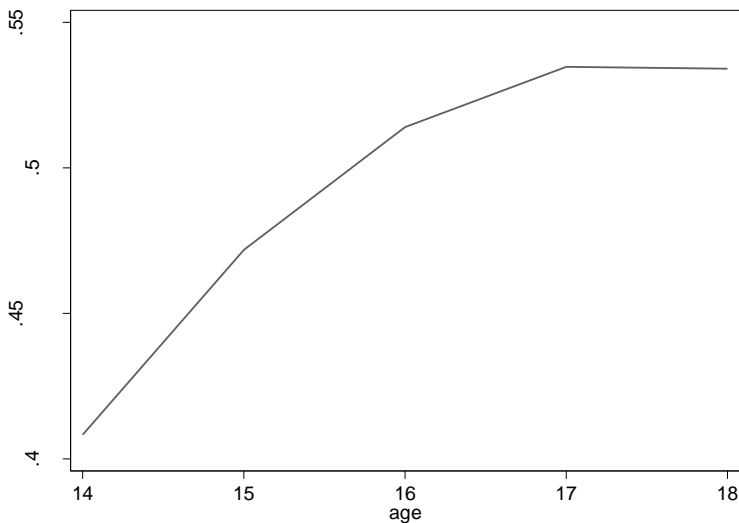
. adjust, gen(pred)

Dependent variable: attit      Equation: attit      Command: xtmixed  
Created variable: pred  
Variables left as is: age16, age16sq

All	xb
	.492161

Key: xb = Linear Prediction

. line pred age, sort



This is identical to calculating:

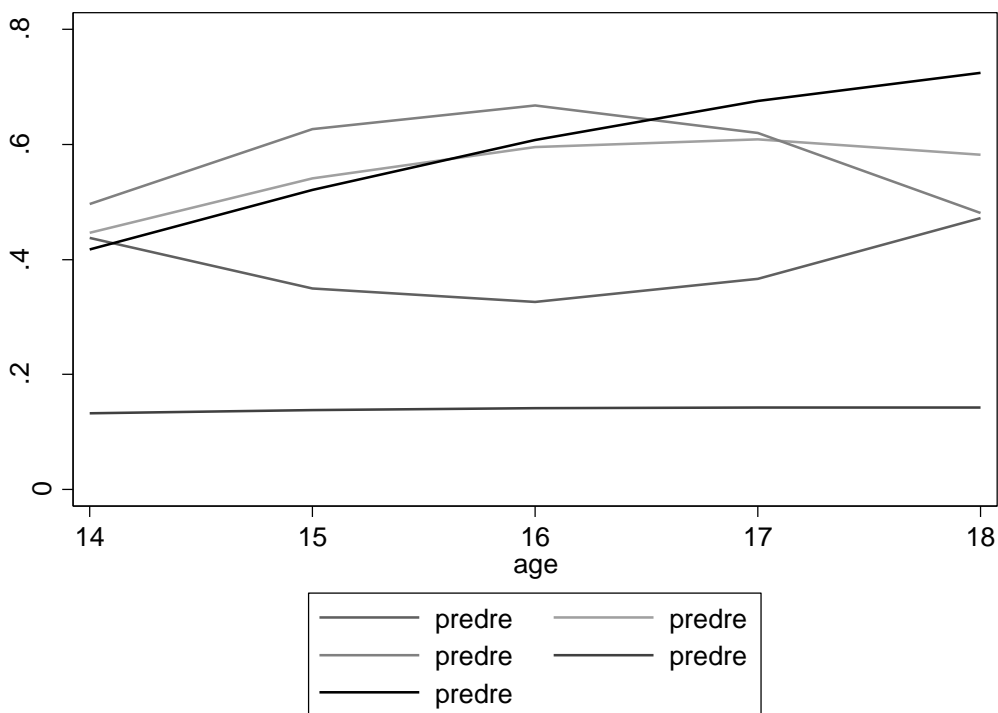
```
. gen pred= .5140183+.0314627 *age16 -.0106962 *age16sq
```

This is the average trajectory; let's see some of the variation across individuals, however. For that, we will obtain estimates of random effects for all three components of the equation and add them to the average coefficients:

```
. predict re*, reffects
```

```
. gen predre=.5140183+re3+(.0314627+re1) *age16 +(-.0106962+re2) *age16sq
```

```
. graph twoway (line predre age if id==7) (line predre age if id==54) (line predre age if id==84) (line predre age if id==104) (line predre age if id==111)
```



Let's compare the linear and quadratic models using LR test and BIC. In order to do use LR test, we should reestimate the model using MLE rather than REML. REML is the default estimation method; REML and ML produce similar regression coefficients, but they differ in terms of estimating the variance components -- if the number of level-2 units is small, then ML variance estimates will be smaller than REML and significance tests based on ML will be biased. When we want to use likelihood ratio tests, however, we might have to opt for ML. When fixed effects are the same and one model has fewer random effects than the other, then both REML or ML may be used for LR test (models have to be nested). When one model has fewer fixed effects (and possibly fewer random effects) than the other, then we have to use ML, which is the case here.

```
. xtmixed attit age16 age16sq || id: age16 age16sq, cov(unstructured) mle
```



```

Iteration 0: log likelihood = 38.418443 (not concave)
Iteration 1: log likelihood = 39.484174 (not concave)
Iteration 2: log likelihood = 40.06454
Iteration 3: log likelihood = 40.952084 (not concave)
Iteration 4: log likelihood = 42.424539 (not concave)
Iteration 5: log likelihood = 43.623775
Iteration 6: log likelihood = 56.205781
Iteration 7: log likelihood = 57.213181
Iteration 8: log likelihood = 57.441535
Iteration 9: log likelihood = 57.442108
Iteration 10: log likelihood = 57.442108

```

Computing standard errors:

```

Mixed-effects ML regression      Number of obs      =      1066
Group variable: id              Number of groups   =       241

                                Obs per group: min =        1
                                avg =         4.4
                                max =         5
                                Wald chi2(1)    =       36.73
                                Prob > chi2    =       0.0000

Log likelihood = 57.442108

```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age	.0323534	.0053383	6.06	0.000	.0218905	.0428164
_cons	-.0243373	.0870451	-0.28	0.780	-.1949426	.1462679

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]	
id: Unstructured				
sd(age)	.0556908	.0057146	.0455448	.068097
sd(_cons)	.9323572	.0913328	.7694838	1.129705
corr(age, _cons)	-.9736444	.0059574	-.9830966	-.9590158
sd(Residual)	.169495	.0048755	.1602035	.1793252

LR test vs. linear regression: chi2(3) = 438.92 Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. lrtest squared .

```

Likelihood-ratio test          LR chi2(4) =      37.53
(Assumption: . nested in squared) Prob > chi2 =      0.0000

```

Note: The reported degrees of freedom assumes the null hypothesis is not on the boundary of the parameter space. If this is not true, then the reported test is conservative.

. estat ic

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	1066	.	57.44211	6	-102.8842	-73.0542

Note: N=Obs used in calculating BIC; see [R] BIC note

Both LR test and difference in BIC (almost 10) indicate that the model with age squared offers a better fit.

Next, let's add variables that could explain variation in attitudes. We start with time-varying (level 1) variables – here we have expo. But it is possible for effects of this variable to also vary across individuals so we allow for such variation:

```
. xtmixed attit age16 age16sq expo || id: age16 age16sq expo, cov(unstructured)
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0: log restricted-likelihood = 191.16984
Iteration 1: log restricted-likelihood = 191.56353
Iteration 2: log restricted-likelihood = 191.56453
Iteration 3: log restricted-likelihood = 191.56453
```

Computing standard errors:

```
Mixed-effects REML regression          Number of obs      =      1066
Group variable: id                    Number of groups   =       241

Obs per group: min =          1
                  avg =         4.4
                  max =          5
```

```
Log restricted-likelihood = 191.56453      Wald chi2(3)      =      269.83
                                          Prob > chi2       =      0.0000
```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
age16	.0229438	.0048663	4.71	0.000	.0134061 .0324816
age16sq	-.0045771	.0032443	-1.41	0.158	-.0109357 .0017816
expo	.4392177	.0303382	14.48	0.000	.3797559 .4986794
_cons	.2522643	.0215611	11.70	0.000	.2100052 .2945234

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]
id: Unstructured			
sd(age16)	.051728	.0051497	.0425584 .0628733
sd(age16sq)	.0261037	.0047828	.0182282 .0373819
sd(expo)	.23522	.0366039	.1733861 .3191056
sd(_cons)	.2071172	.0215611	.1688905 .2539962
corr(age16, age16sq)	-.2236336	.1771147	-.5319718 .1370675
corr(age16, expo)	-.183421	.1671742	-.4812296 .1523468
corr(age16, _cons)	.1768226	.1443242	-.1128169 .4387654
corr(age16sq, expo)	.1805575	.2097605	-.2377791 .5423907
corr(age16sq, _cons)	-.4224947	.1591735	-.6807377 -.0708434
corr(expo, _cons)	-.6382323	.0978611	-.7927606 -.4066177
sd(Residual)	.1410003	.0053562	.1308836 .151899

```
LR test vs. linear regression:      chi2(10) =      290.82      Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

There is significant variation in slopes of all of these three level 1 variables. Next, we add level 2 (time invariant) variables as predictors of attitudes (but not yet of slopes). We have the following level 2 predictors: female, minority, and income.

```
. xtmixed attit age16 age16sq expo female minority income || id: age16 age16sq expo,
cov(unstructured)
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0: log restricted-likelihood = 187.55152
Iteration 1: log restricted-likelihood = 187.96413
Iteration 2: log restricted-likelihood = 187.96519
Iteration 3: log restricted-likelihood = 187.96519
```

Computing standard errors:

```
Mixed-effects REML regression          Number of obs    =    1066
Group variable: id                    Number of groups  =     241

Obs per group: min =         1
                  avg =        4.4
                  max =         5

Wald chi2(6)          =    290.41
Prob > chi2          =     0.0000

Log restricted-likelihood = 187.96519
```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age16	.0228237	.0048686	4.69	0.000	.0132815	.0323659
age16sq	-.0044062	.0032464	-1.36	0.175	-.010769	.0019566
expo	.4416672	.0300694	14.69	0.000	.3827323	.5006021
female	-.0498218	.0220006	-2.26	0.024	-.0929423	-.0067013
minority	.0223901	.0278347	0.80	0.421	-.0321649	.0769451
income	.0141895	.0048562	2.92	0.003	.0046716	.0237074
_cons	.2082864	.0333672	6.24	0.000	.142888	.2736849

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]	
id: Unstructured				
sd(age16)	.051715	.0051549	.0425372	.062873
sd(age16sq)	.0262153	.0047748	.0183451	.037462
sd(expo)	.2308826	.0363514	.1695796	.3143467
sd(_cons)	.1971736	.0217087	.1589029	.2446615
corr(age16, age16sq)	-.2243084	.1763973	-.5315029	.1350313
corr(age16, expo)	-.1753755	.1690745	-.4770072	.1632151
corr(age16, _cons)	.2036548	.1477114	-.0952044	.468837
corr(age16sq, expo)	.1886991	.2106696	-.2328136	.5505275
corr(age16sq, _cons)	-.4387443	.162634	-.6990387	-.0757856
corr(expo, _cons)	-.6140747	.1078263	-.7836296	-.3593735
sd(Residual)	.1411289	.0053587	.1310074	.1520323

```
LR test vs. linear regression:      chi2(10) =    274.50   Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

Since we are now trying to model variance in the constant (intercept), we should make sure that intercept meaningful by making 0 a meaningful value on all predictors. Dummies are ok as long as they are coded 0/1 but continuous predictors should be mean-centered.

```
. for var expo income: sum X \ gen Xm=X-r(mean)

-> sum expo

-----+-----
Variable |      Obs      Mean   Std. Dev.   Min     Max
-----+-----
expo    |     1066   .5601501   .3106114     0     1.61

-> gen expom=expo-r(mean)
(139 missing values generated)

-> sum income

-----+-----
Variable |      Obs      Mean   Std. Dev.   Min     Max
-----+-----
income  |     1205   4.091286   2.346617     1     10

-> gen incomem=income-r(mean)

. xtmixed attit age16 age16sq expom female minority incomem || id: age16 age16sq
expom, cov(unstructured)
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0: log restricted-likelihood = 186.94021
Iteration 1: log restricted-likelihood = 187.96071
Iteration 2: log restricted-likelihood = 187.96519
Iteration 3: log restricted-likelihood = 187.96519
```

Computing standard errors:

```
Mixed-effects REML regression          Number of obs   =    1066
Group variable: id                    Number of groups =     241

                                         Obs per group: min =      1
                                         avg =                4.4
                                         max =                5

                                         Wald chi2(6)     =    290.41
Log restricted-likelihood = 187.96519    Prob > chi2      =     0.0000
```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age16	.0228237	.0048686	4.69	0.000	.0132815	.0323659
age16sq	-.0044062	.0032464	-1.36	0.175	-.010769	.0019566
expom	.4416672	.0300694	14.69	0.000	.3827323	.5006021
female	-.0498218	.0220006	-2.26	0.024	-.0929423	-.0067013
minority	.0223901	.0278347	0.80	0.421	-.0321649	.0769451
incomem	.0141895	.0048562	2.92	0.003	.0046716	.0237074
_cons	.5137396	.0178053	28.85	0.000	.4788418	.5486374

```
-----+-----
Random-effects Parameters | Estimate Std. Err. [95% Conf. Interval]
```

```

-----+-----
id: Unstructured |
      sd(age16) | .051715 .0051549 .0425372 .062873
      sd(age16sq) | .0262153 .0047748 .0183451 .037462
      sd(expom) | .2308826 .0363514 .1695796 .3143468
      sd(_cons) | .1558374 .0120352 .1339472 .1813049
corr(age16, age16sq) | -.2243084 .1763973 -.5315029 .1350313
  corr(age16, expom) | -.1753755 .1690743 -.4770069 .1632147
  corr(age16, _cons) | .1121311 .1200113 -.124952 .3371005
corr(age16sq, expom) | .1886991 .2106692 -.2328129 .550527
corr(age16sq, _cons) | -.3985215 .1260241 -.6141318 -.1275533
  corr(expom, _cons) | .0529374 .156538 -.2493168 .3457936
-----+-----
      sd(Residual) | .1411289 .0053587 .1310074 .1520323
-----+-----
LR test vs. linear regression:      chi2(10) = 274.50 Prob > chi2 = 0.0000

```

Note: LR test is conservative and provided only for reference.

The kind of centering we just applied is called grand-mean centering. The centering issue is important in mixed models.

*Centering choices for time-varying (level-1) predictors:*

### 1. Natural metric (X):

You should only use the original metric if the value of 0 for a predictor is a meaningful value. When 0 is not meaningful, the estimate of the intercept will be arbitrary and may be estimated with poor precision. Lack of precision in mixed models can be very problematic. First, because you are estimating within-group intercepts, thus with possibly small N, the estimates may be quite unstable. Second, because you may be trying to model variation in these intercepts, your model will be affected by the unreliability of the estimates.

### 2. Grand-mean centering (X - grand mean):

This will address the problems with estimation of intercept in original metric. Because the 0 values will fall in the middle of the distribution of the predictors, the intercept estimates will be estimated with much more precision. The intercept is also interpretable. Specifically, it will represent the value for a person with a (grand) average on every predictor. The interpretation of the intercepts is now “adjusted group mean.” The interpretation of slopes does not change. So we can interpret the fixed effect for the intercept as the average attitudes value adjusted for exposure – i.e., the average attitudes level for someone with average exposure to deviant peers.

Note that while it may seem inappropriate at first to center a dummy variable, in mixed models it can actually be quite useful. If uncentered, the intercept in a model with a dummy variable is the average value when the dummy variable is 0. If the dummy variable is centered, the intercept then becomes the mean adjusted for the proportion of time points with the dummy variable=1, so essentially it is the mean for an average case. We would only consider centering dummy variables when we would like to treat them as controls rather than main predictors of interest.

### 3. Group-mean centering (X – group mean):

Predictors can also be centered around the mean value for a given person (averaged over time). Recall how we used group-mean centered variables to indicate the change component within random effects models along with group means to indicate cross-sectional effects of differences

across individuals. The intercept can then be interpreted as the average outcome for each person. This allows interpretation of parameter estimates as effects of change over time within-person. Under grand-mean centering or no centering, the parameter estimates reflect a combination of change over time and differences across individuals. But when we use a group-centered predictor, we only estimate only change effects (within-person component). In order not to discard the effects of differences across individuals, we should include person level variables alongside group-mean centered predictors. This is a common way to separate within and between unit effects in mixed effects model (we did that in random effects model as well):

```
. by id: egen expomean=mean(expo)

. gen expochange=expo-expomean
(139 missing values generated)

. xtmixed attit age16 age16sq expochange expomean female minority incomem || id:
age16 age16sq expochange, cov(unstructured)
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0: log restricted-likelihood = 198.38935
Iteration 1: log restricted-likelihood = 199.73882
Iteration 2: log restricted-likelihood = 199.74883
Iteration 3: log restricted-likelihood = 199.74883
```

Computing standard errors:

```
Mixed-effects REML regression          Number of obs      =      1066
Group variable: id                    Number of groups   =       241

Obs per group: min =          1
                  avg =         4.4
                  max =          5

Log restricted-likelihood = 199.74883   Wald chi2(7)       =      378.11
                                      Prob > chi2         =       0.0000
```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
age16	.0249635	.0049267	5.07	0.000	.0153073 .0346196
age16sq	-.005833	.0032221	-1.81	0.070	-.0121481 .0004821
expochange	.3468658	.0373676	9.28	0.000	.2736266 .420105
expomean	.6187866	.0411992	15.02	0.000	.5380376 .6995355
female	-.0433344	.0214128	-2.02	0.043	-.0853027 -.001366
minority	.0118263	.0269929	0.44	0.661	-.0410789 .0647315
incomem	.0165189	.004742	3.48	0.000	.0072248 .0258131
_cons	.1733223	.0293303	5.91	0.000	.1158361 .2308086

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]
id: Unstructured			
sd(age16)	.052596	.0051691	.0433808 .0637688
sd(age16sq)	.0258875	.0047426	.018078 .0370707
sd(expoch~e)	.2647982	.0447321	.1901614 .3687293
sd(_cons)	.1608366	.0109195	.1407977 .1837276

```

corr(age16,age16sq) | -.2223735   .1773293   -.531183   .1385906
corr(age16,expoch~e) | -.1868188   .1761984   -.4981708   .1672075
corr(age16,_cons) | .142767   .1130265   -.0822021   .353892
corr(age16sq,expoch~e) | .0713353   .2202061   -.3472796   .4662441
corr(age16sq,_cons) | -.418331   .1173367   -.6196434   -.1653758
corr(expoch~e,_cons) | .1592605   .1630533   -.1657248   .4530434
-----+-----
sd(Residual) | .1383106   .0052878   .1283254   .1490727
-----+-----
LR test vs. linear regression:      chi2(10) =    275.91   Prob > chi2 = 0.0000

```

Note: LR test is conservative and provided only for reference.

**We can compare that model to a model with grand-mean centered expo variable and level 2 average expomean variable:**

```
. xtmixed attit age16 age16sq expom expomean female minority incomem || id: age16
age16sq expom, cov(unstructured)
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0: log restricted-likelihood = 197.17804
Iteration 1: log restricted-likelihood = 198.21598
Iteration 2: log restricted-likelihood = 198.22082
Iteration 3: log restricted-likelihood = 198.22082
```

Computing standard errors:

```
Mixed-effects REML regression      Number of obs      =      1066
Group variable: id                 Number of groups   =       241

Obs per group: min =              1
                  avg =              4.4
                  max =              5

Wald chi2(7) = 330.16
Log restricted-likelihood = 198.22082   Prob > chi2 = 0.0000
```

```
-----+-----
attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
age16 |   .02526   .0049173     5.14  0.000   .0156222   .0348977
age16sq | -.0057001  .0032401    -1.76  0.079  -.0120506   .0006504
expom |   .34251   .0353469     9.69  0.000   .2732314   .4117887
expomean | .2747236   .0538598     5.10  0.000   .1691603   .3802868
female |  -.042497  .0213384    -1.99  0.046  -.0843195  -.0006746
minority | .0158907  .0269545     0.59  0.556  -.0369392   .0687206
incomem | .0154528  .0047072     3.28  0.001   .0062268   .0246787
_cons |   .3621374  .0347338    10.43  0.000   .2940603   .4302145
-----+-----
```

```
-----+-----
Random-effects Parameters |      Estimate   Std. Err.     [95% Conf. Interval]
-----+-----
id: Unstructured
sd(age16) |   .0528287   .005093     .043733   .0638161
sd(age16sq) | .0264583   .004667     .0187249   .0373858
sd(expom) | .2146471   .0364519     .1538764   .2994183
sd(_cons) | .1523349   .0116032     .1312092   .176862
-----+-----
```

corr(age16, age16sq)	-.2216727	.1706863	-.520658	.1257437
corr(age16, expom)	-.1293797	.1741297	-.4440041	.2136465
corr(age16, _cons)	.1794464	.117767	-.0570264	.3968543
corr(age16sq, expom)	.1275558	.216884	-.2948421	.508252
corr(age16sq, _cons)	-.4308474	.1207651	-.6360936	-.1686615
corr(expom, _cons)	.0183717	.1589695	-.2851748	.3185689
-----				
sd(Residual)	.1400249	.0052544	.1300961	.1507114
-----				
LR test vs. linear regression:	chi2(10) =	272.86	Prob > chi2 =	0.0000

Note: LR test is conservative and provided only for reference.

### *Level-2 predictors:*

Centering issues for level-2 predictors are essentially the same issues faced in any regression. If the value of 0 for a predictor is not meaningful, the intercept will not have a meaningful interpretation and the estimate may lack precision. When these conditions exist, grand-mean centering is advisable.

Next, we will estimate a model where we will use cross-level interactions to explain variance in slopes across individuals. That is, we will introduce interactions of level 1 predictors with level 2 time-invariant variables and then see what happens to variance of slopes of those level 1 predictors.

```
. for var expomean female minority incomem: gen age16X=age16*X \ gen
age16sqX=age16sq*X \ gen expochX=expochange*X

-> gen age16expomean=age16*expomean

-> gen age16sqexpomean=age16sq*expomean

-> gen expochexpomean=expochange*expomean
(139 missing values generated)

-> gen age16female=age16*female

-> gen age16sqfemale=age16sq*female

-> gen expochfemale=expochange*female
(139 missing values generated)

-> gen age16minority=age16*minority

-> gen age16sqminority=age16sq*minority

-> gen expochminority=expochange*minority
(139 missing values generated)

-> gen age16incomem=age16*incomem

-> gen age16sqincomem=age16sq*incomem

-> gen expochincomem=expochange*incomem
(139 missing values generated)

. xtmixed attit age16 age16expomean age16female age16minority age16incomem age16sq
age16sqexpomean age16sqfemale age16sqminority age16sqincomem expochange
```

```

expochexpomean expochfemale expochminority expochincomem expomean female minority
incomem || id: age16 age16sq expochange , cov(unstructured)

```

Performing EM optimization:

Performing gradient-based optimization:

```

Iteration 0: log restricted-likelihood = 173.9332
Iteration 1: log restricted-likelihood = 175.61856
Iteration 2: log restricted-likelihood = 175.63863
Iteration 3: log restricted-likelihood = 175.63863

```

Computing standard errors:

```

Mixed-effects REML regression      Number of obs      =      1066
Group variable: id                 Number of groups   =       241

Obs per group: min =              1
                  avg =              4.4
                  max =              5

```

```

Log restricted-likelihood = 175.63863      Wald chi2(19)      =      426.81
                                          Prob > chi2        =      0.0000

```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age16	.0493045	.0132632	3.72	0.000	.023309	.0752999
age16expom~n	-.0458508	.0191619	-2.39	0.017	-.0834075	-.008294
age16female	.0047919	.0099519	0.48	0.630	-.0147135	.0242972
age16minor~y	-.0073418	.0126937	-0.58	0.563	-.0322211	.0175375
age16incomem	-.0024325	.0021744	-1.12	0.263	-.0066942	.0018292
age16sq	-.0105266	.0088051	-1.20	0.232	-.0277844	.0067311
age16sqexp~n	-.0042772	.0128712	-0.33	0.740	-.0295042	.0209499
age16sqfem~e	.0149019	.0065016	2.29	0.022	.0021589	.0276449
age16sqmin~y	.0047221	.0083582	0.56	0.572	-.0116597	.0211039
age16sqinc~m	.0021889	.0014118	1.55	0.121	-.0005782	.0049561
expochange	.2847031	.1123478	2.53	0.011	.0645054	.5049008
expochexpo~n	.2296443	.1593827	1.44	0.150	-.08274	.5420287
expochfemale	-.0139817	.0758856	-0.18	0.854	-.1627148	.1347513
expochmino~y	-.3022909	.0927468	-3.26	0.001	-.4840713	-.1205104
expochinco~m	-.044656	.0171895	-2.60	0.009	-.0783468	-.0109653
expomean	.6238642	.0495817	12.58	0.000	.5266859	.7210426
female	-.0740025	.0255484	-2.90	0.004	-.1240765	-.0239285
minority	-.0078201	.0322398	-0.24	0.808	-.0710089	.0553687
incomem	.0102604	.0056377	1.82	0.069	-.0007893	.0213102
_cons	.1869384	.034303	5.45	0.000	.1197058	.2541711

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]	
id: Unstructured				
sd(age16)	.0519192	.0052175	.0426372	.063222
sd(age16sq)	.0245313	.0048939	.0165924	.0362687
sd(expoc~ge)	.2452769	.0440905	.1724433	.3488727
sd(_cons)	.1596776	.0108551	.1397585	.1824356
corr(age16,age16sq)	-.2635438	.1876893	-.5818363	.1247552
corr(age16,expoc~ge)	-.2494407	.1852378	-.5662355	.1315645
corr(age16,_cons)	.1554511	.1139626	-.072047	.3675742
corr(age16sq,expoc~ge)	.2379141	.2394154	-.249461	.6291221
corr(age16sq,_cons)	-.4062904	.1228207	-.6165855	-.1418793

```

      corr(expoc~ge, _cons) |   .0437171   .1729702   -.287574   .3656647
-----+-----
      sd(Residual) |   .1383646   .0052703   .1284112   .1490895
-----+-----
LR test vs. linear regression:      chi2(10) =   278.69   Prob > chi2 = 0.0000

```

Note: LR test is conservative and provided only for reference.

**Let's simplify the model by omitting non-significant cross-level interactions; we will use LR test and BIC to make sure we do not omit anything important:**

```

. xtmixed attit age16 age16expomean age16female age16sq age16sqexpomean
age16sqfemale expochange expochminority expochincomem expomean female minority
incomem || id: age16 age16sq expochange , cov(unstructured) mle

```

Performing EM optimization:

Performing gradient-based optimization:

```

Iteration 0:   log likelihood = 239.02435
Iteration 1:   log likelihood = 240.79649
Iteration 2:   log likelihood = 240.81527
Iteration 3:   log likelihood = 240.81527

```

Computing standard errors:

```

Mixed-effects ML regression              Number of obs   =   1066
Group variable: id                      Number of groups =    241

Obs per group: min =     1
                  avg =    4.4
                  max =     5

Wald chi2(13)   =   430.09
Prob > chi2     =   0.0000

Log likelihood = 240.81527

```

```

-----+-----
      attit |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
      age16 |   .0448121   .0125932     3.56  0.000   .0201299   .0694943
age16expom~n | -.0408274   .0187166    -2.18  0.029  -.0775112  -.0041436
  age16female |   .0039725   .0095473     0.42  0.677  -.0147399   .0226849
   age16sq | -.0071441   .0083618    -0.85  0.393  -.0235329   .0092447
age16sqexp~n | -.0091057   .0124199    -0.73  0.463  -.0334482   .0152368
age16sqfem~e |   .0151555   .0062652     2.42  0.016   .0028759   .0274351
  expochange |   .4191731   .0422462     9.92  0.000   .3363721   .5019741
expochmino~y | -.3121521   .0882582    -3.54  0.000  -.4851349  -.1391693
expochinco~m | -.0550901   .016154    -3.41  0.001  -.0867514  -.0234288
  expomean |   .6318958   .049011    12.89  0.000   .535836   .7279556
   female | -.0738328   .0252351    -2.93  0.003  -.1232926  -.024373
  minority |   .0038862   .0267865     0.15  0.885  -.0486143   .0563868
  incomem |   .0152012   .0047064     3.23  0.001   .0059768   .0244256
   _cons |   .1799113   .0336683     5.34  0.000   .1139226   .2458999
-----+-----

```

Interpreting fixed effects:

The effect of age is best understood graphically.

Effect of change in exposure:

For a non-minority with average income, one unit change in exposure is associated with .42 increase in attitudes.



```

age16minor~y | -.0073877 .0124986 -0.59 0.554 -.0318845 .017109
age16incomem | -.0024181 .0021411 -1.13 0.259 -.0066145 .0017784
    age16sq | -.0105712 .0086834 -1.22 0.223 -.0275903 .006448
age16sqexp~n | -.0041202 .0126952 -0.32 0.746 -.0290023 .0207619
age16sqfem~e | .0148405 .0064114 2.31 0.021 .0022744 .0274066
age16sqmin~y | .0047483 .0082399 0.58 0.564 -.0114016 .0208982
age16sqinc~m | .0022079 .0013923 1.59 0.113 -.0005208 .0049367
    expchange | .2869866 .110384 2.60 0.009 .0706379 .5033353
expochexpo~n | .2282904 .156432 1.46 0.144 -.0783107 .5348915
expochfemale | -.0151889 .0744531 -0.20 0.838 -.1611144 .1307366
expochmino~y | -.3040498 .0908928 -3.35 0.001 -.4821965 -.1259031
expochinco~m | -.0445934 .0168649 -2.64 0.008 -.077648 -.0115387
    expomean | .6235837 .04901 12.72 0.000 .5275259 .7196416
    female | -.0738721 .0252524 -2.93 0.003 -.123366 -.0243783
    minority | -.0079024 .0318651 -0.25 0.804 -.0703567 .054552
    incomem | .0102222 .0055722 1.83 0.067 -.000699 .0211435
    _cons | .187023 .0339053 5.52 0.000 .12057 .2534761
-----

```

```

-----
Random-effects Parameters | Estimate Std. Err. [95% Conf. Interval]
-----+-----
id: Unstructured
    sd(age16) | .0506134 .0051428 .041474 .0617668
    sd(age16sq) | .0234736 .0049506 .015526 .0354895
    sd(expoc~ge) | .2334688 .0436144 .1618883 .3366992
    sd(_cons) | .1571853 .0106749 .1375955 .1795641
    corr(age16, age16sq) | -.2765475 .1934959 -.6009231 .1260373
    corr(age16, expoc~ge) | -.2525177 .1898236 -.5753521 .1383928
    corr(age16, _cons) | .1594468 .1142585 -.0688563 .3718804
    corr(age16sq, expoc~ge) | .2475761 .2498039 -.2624586 .6494836
    corr(age16sq, _cons) | -.4012455 .126131 -.6167747 -.1297468
    corr(expoc~ge, _cons) | .0476106 .1768763 -.2911408 .3757532
-----
    sd(Residual) | .1383681 .0052563 .1284401 .1490635
-----

```

```
LR test vs. linear regression:      chi2(10) = 278.01 Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

```
. lrtest . reduced
```

```

Likelihood-ratio test
(Assumption: reduced nested in .)
LR chi2(6) = 5.55
Prob > chi2 = 0.4754

```

```
. estat ic
```

```

-----
Model | Obs ll(null) ll(model) df AIC BIC
-----+-----
. | 1066 . 243.5905 31 -425.1811 -271.0594
-----

```

Note: N=Obs used in calculating BIC; see [R] BIC note

No significant difference in model fit indicated by LR test, and BIC is substantially smaller in the reduced model; therefore, we can use the reduced model.

To summarize model building in mixed effects models, we have a number of options:

- The effects of level 1 predictors can be estimated as either fixed effects or random effects
- Level 2 predictors can be used to predict the intercept (i.e., as direct predictors of DV)
- Level 2 predictors can explain the variation in slopes of level 1 predictors (i.e., as cross-level interactions)

Because so many components are involved, it is best to proceed incrementally.

1. Start by fitting a model with only the time variable. Evaluate level 2 variance in intercepts and time slopes to see if a mixed effects model is necessary.
2. Estimate a model with random intercept and slopes using only level 1 variables (all slopes should be random effects). Evaluate slope variance and decide whether some slopes should be fixed (i.e., no random component included for it).
3. Estimate a model with both level 1 variables and level 2 variables used as predictors of intercepts.
4. For slopes with significant variance, use level 2 predictors to explain that variance (i.e., estimate a model with cross-level interactions).
5. If the slope variance remaining after entering level 2 predictors is not statistically significant, estimate that slope as non-randomly varying (i.e., keep cross-level interactions but do not include a random component for that slope).
6. When making decisions what variables to include and whether to estimate random or fixed effects, use LR tests and BIC values to select a model with best fit and parsimony.

Next, let's explore how much variance in slopes of each of level 1 predictors our final model explain. For that, let's compare it to a model without any level 2 predictors:

```
. xtmixed attit age16 age16sq expochange || id: age16 age16sq expochange,
cov(unstructured)
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0: log restricted-likelihood = 127.87781
Iteration 1: log restricted-likelihood = 129.02377
Iteration 2: log restricted-likelihood = 129.0304
Iteration 3: log restricted-likelihood = 129.0304
```

Computing standard errors:

```
Mixed-effects REML regression          Number of obs      =      1066
Group variable: id                    Number of groups   =       241

Obs per group: min =          1
                  avg =         4.4
                  max =          5
```

```
Log restricted-likelihood = 129.0304      Wald chi2(3)       = 135.39
                                          Prob > chi2        = 0.0000
```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
age16	.0244345	.0049234	4.96	0.000	.0147849 .0340841
age16sq	-.00557	.003245	-1.72	0.086	-.01193 .0007901
expochange	.3406091	.0375955	9.06	0.000	.2669232 .414295
_cons	.5031748	.0164454	30.60	0.000	.4709425 .5354071



```

    age16sq | -.0071184 .0084317 -0.84 0.399 -.0236442 .0094073
age16sqexp~n | -.0091831 .0125238 -0.73 0.463 -.0337293 .0153631
age16sqfem~e | .0151793 .0063176 2.40 0.016 .002797 .0275616
    expochange | .4184038 .0427378 9.79 0.000 .3346392 .5021684
expochmino~y | -.3111393 .089373 -3.48 0.000 -.4863071 -.1359714
expochinco~m | -.0551251 .0163477 -3.37 0.001 -.087166 -.0230842
    expomean | .6320613 .0494977 12.77 0.000 .5350476 .7290749
    female | -.0738951 .0254879 -2.90 0.004 -.1238505 -.0239397
    minority | .0038667 .0270767 0.14 0.886 -.0492027 .0569361
    incomem | .0151963 .0047581 3.19 0.001 .0058707 .024522
    _cons | .1798652 .0340044 5.29 0.000 .1132177 .2465127
-----

```

```

-----
Random-effects Parameters | Estimate Std. Err. [95% Conf. Interval]
-----+-----
id: Unstructured
    sd(age16) | .051896 .0051872 .0426632 .0631269
    sd(age16sq) | .0245891 .0048454 .0167112 .0361807
    sd(expoc~ge) | .2465608 .0435606 .1743974 .3485845
    sd(_cons) | .1601864 .0108543 .1402646 .1829378
    corr(age16, age16sq) | -.2924035 .1859938 -.6042509 .0971238
    corr(age16, expoc~ge) | -.2427525 .1829635 -.5571936 .1325751
    corr(age16, _cons) | .1587813 .1132003 -.0673683 .3694128
    corr(age16sq, expoc~ge) | .256062 .2371067 -.2311838 .6406166
    corr(age16sq, _cons) | -.4168253 .1210471 -.6236643 -.1554373
    corr(expoc~ge, _cons) | .044171 .171939 -.2853101 .3643155
-----+-----
    sd(Residual) | .1383125 .0052534 .1283899 .149002
-----

```

LR test vs. linear regression:      chi2(10) =    280.32    Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

. estat recov

Random-effects covariance matrix for level id

```

-----+-----
    |      age16      age16sq  expocha~e      _cons
-----+-----
age16 | .0026932
age16sq | -.0003731 .0006046
expochange | -.0031061 .0015524 .0607922
    _cons | .00132 -.0016418 .0017446 .0256597
-----

```

To see both matrices back to back, I copied the one from the model above:

```

-----+-----
    |      age16      age16sq  expocha~e      _cons
-----+-----
age16 | .0027331
age16sq | -.0002799 .0006961
expochange | -.0029392 .0001914 .0736745
    _cons | -.0009538 -.0020919 .0143702 .052996
-----

```

So the variance in age16 slope went from .0027331 to .0026932, variance in age16sq slope from .0006961 to .0006046, variance in expochange slope from .0736745 to .0607922, and variance in intercepts from .052996 to .0256597. We can calculate all of these as percentages:

```

. di (.0027331-.0026932)/.0027331
.01459881

```

```
. di (.0006961-.0006046)/.0006961
.13144663

. di (.0736745-.0607922)/.0736745
.17485426

. di (.052996-.0256597)/.052996
.51581817
```

We might also want to know overall how much of within-person and between-person variance we are explaining here. Typically, in growth curve models, we compare our full model to a model with only time variables (in our case, age and age squared) that does not include any random slopes, only random intercept. Here's our null model:

```
. xtmixed attit age16 age16sq || id:, variance
```

Performing EM optimization:

Performing gradient-based optimization:

```
Iteration 0: log restricted-likelihood = 28.467641
Iteration 1: log restricted-likelihood = 28.467641
```

Computing standard errors:

```
Mixed-effects REML regression          Number of obs    =    1066
Group variable: id                     Number of groups =     241

Obs per group: min =      1
                avg =     4.4
                max =      5

Log restricted-likelihood = 28.467641    Wald chi2(2)     =     67.16
                                         Prob > chi2      =     0.0000
```

```
-----+-----
      attit |      Coef.   Std. Err.      z    P>|z|    [95% Conf. Interval]
-----+-----
      age16 |   .0322153   .0042392     7.60  0.000    .0239066   .040524
  age16sq |  -.0103421   .0034931    -2.96  0.003   -.0171885  -.0034958
    _cons |   .5130908   .0163918    31.30  0.000    .4809634   .5452181
-----+-----
```

```
-----+-----
Random-effects Parameters | Estimate   Std. Err.    [95% Conf. Interval]
-----+-----
id: Identity              |
      var(_cons) |   .0445029   .0049118    .035846   .0552504
-----+-----
      var(Residual) |   .0358148   .0017688    .0325105   .0394549
-----+-----
```

```
LR test vs. linear regression: chibar2(01) = 400.32 Prob >= chibar2 = 0.0000
```

```
. xtmixed attit age16 age16expomean age16female age16sq age16sqexpomean
age16sqfemale expchange expochminority expochincomem expomean female minority
incomem || id:, variance
```

Performing EM optimization:

Performing gradient-based optimization:



We can also run xtreg to obtain approximate R-squared values (since we do not need random slopes for that) but those R squared values will compare our model to a null model without any variables (so without age variables as well), so they will be somewhat different:

```
. xtreg attit age16 age16expomean age16female age16sq age16sqexpomean age16sqfemale
expochange expochminority expochincomem expomean female minority incomem
```

```
Random-effects GLS regression           Number of obs   =       1066
Group variable: id                     Number of groups =        241

R-sq:  within = 0.2477                 Obs per group:  min =         1
        between = 0.5058                    avg =         4.4
        overall = 0.4028                    max =         5
```

```
Random effects u_i ~ Gaussian          Wald chi2(13)    =       504.52
corr(u_i, X) = 0 (assumed)            Prob > chi2     =       0.0000
```

attit	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age16	.0443167	.0103721	4.27	0.000	.0239877	.0646457
age16expom~n	-.0426375	.0153122	-2.78	0.005	-.0726489	-.0126261
age16female	.0039313	.0077485	0.51	0.612	-.0112554	.0191181
age16sq	-.0077005	.0085027	-0.91	0.365	-.0243655	.0089645
age16sqexp~n	-.0071247	.0126492	-0.56	0.573	-.0319167	.0176674
age16sqfem~e	.0148111	.0063908	2.32	0.020	.0022854	.0273368
expochange	.4450367	.0360808	12.33	0.000	.3743196	.5157538
expochmino~y	-.3099835	.0741428	-4.18	0.000	-.4553007	-.1646663
expochinco~m	-.0478129	.0141375	-3.38	0.001	-.075522	-.0201038
expomean	.6297261	.0487233	12.92	0.000	.5342302	.7252221
female	-.0724031	.0251082	-2.88	0.004	-.1216142	-.023192
minority	.0022791	.0273332	0.08	0.934	-.0512929	.0558512
incomem	.0146007	.0048037	3.04	0.002	.0051855	.0240159
_cons	.1798357	.033503	5.37	0.000	.114171	.2455004
sigma_u	.14136818					
sigma_e	.17125674					
rho	.40526077	(fraction of variance due to u_i)				

### Diagnostics for mixed effects models

To conduct diagnostics for mixed effects models, we can obtain predicted values and various residuals; here is what we can obtain using predict command:

```
xb          xb, linear predictor for the fixed portion of the model
stdp       standard error of the fixed-portion linear prediction xb
fitted     fitted values, linear predictor of the fixed portion plus
           contributions based on predicted random effects
residuals  residuals, response minus fitted values
rstandard  standardized residuals
reffects   best linear unbiased predictions (BLUPs) of the
           random effects. By default, BLUPs for all random effects in the
           model are calculated. You must specify q new variables, where q is
           the number of random-effects terms in the model.
reses     standard errors of the best linear unbiased predictions (BLUPs) of
           the random effects. By default, standard errors for all BLUPs in
           the model are calculated. You must specify q new variables.
```

Thus, residuals will give you level 1 residuals, and reffects will give you level 2 residuals for each level 2 random component. You should examine both types of residuals to assess normality. To assess linearity, you can plot residuals (level 1 and level 2) against each predictor using lowess (there should be no relationship between them). You can also examine both level 1 and level 2 residuals for potential outliers (i.e., look for observations with large standardized residuals).

There is no robust option available with xtmixed, but to obtain standard errors and significance tests that are less dependent on assumptions, you can use bootstrapping, although it does take time to calculate. E.g., compare to the model on pp.9-10:

```
. bootstrap: xtmixed attit age16 age16sq expo || id: age16 age16sq expo,
cov(unstructured)
(running xtmixed on estimation sample)

Bootstrap replications (50)
-----+----- 1 -----+----- 2 -----+----- 3 -----+----- 4 -----+----- 5
..... 50

Mixed-effects REML regression                Number of obs      =       1066
Group variable: id                          Number of groups   =        241

                                           Obs per group: min =         1
                                           avg =                 4.4
                                           max =                 5

                                           Wald chi2(3)       =       189.40
Log restricted-likelihood = 191.56453        Prob > chi2        =        0.0000

-----+-----
      |      Observed      |      Bootstrap      |      Normal-based      |
      |      Coef.         |      Std. Err.      |      [95% Conf. Interval] |
-----+-----
age16 |      .0229438      |      .0048823      |      4.70  0.000      |      .0133748      .0325129
age16sq |     -.0045771     |      .004397       |     -1.04  0.298      |     -.013195     .0040408
expo   |      .4392177     |      .0425567     |     10.32  0.000      |      .355808     .5226273
_cons  |      .2522643     |      .0285277     |      8.84  0.000      |      .1963511     .3081776
-----+-----

Random-effects Parameters      |      Observed      |      Bootstrap      |      Normal-based      |
      |      Estimate      |      Std. Err.      |      [95% Conf. Interval] |
-----+-----
id: Unstructured
      |      |      |      |      |
      | sd(age16) |      .051728      |      .005318      |      .042288      .0632754
      | sd(age16sq) |      .0261037     |      .0025442     |      .0215645     .0315984
      | sd(expo) |      .23522       |      .0389185     |      .170074      .3253199
      | sd(_cons) |      .2071172     |      .0307767     |      .1547859     .2771412
      | corr(age16,age16sq) |     -.2236336     |      .1263644     |     -.4527754     .0332182
      | corr(age16,expo) |     -.183421     |      .1352113     |     -.4298852     .0884828
      | corr(age16,_cons) |      .1768226     |      .1391072     |     -.102384     .4302029
      | corr(age16sq,expo) |      .1805575     |      .1465234     |     -.1138044     .4457768
      | corr(age16sq,_cons) |     -.4224947     |      .1118236     |     -.6153697     -.1818851
      | corr(expo,_cons) |     -.6382323     |      .0788718     |     -.7682403     -.4576631
-----+-----
      | sd(Residual) |      .1410003     |      .0239871     |      .1010213     .1968009
-----+-----

LR test vs. linear regression:      chi2(10) =      290.82      Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

Overall, when preparing to estimate mixed effects models and conducting diagnostics, it is often useful to look separately at level 1 and level 2 datasets. Do not forget that preliminary data examination stage – it is important! Level 1 variables can be explored in the original dataset, but to examine linearity, normality, and univariate outliers for level 2 variables, it is useful to create level 2 dataset:

```
. keep id attit female minority income incomem expomean
. by id: egen attitmean=mean(attit)
. drop attit
. by id: keep if _n==_N
(964 observations deleted)
. sum id
```

Variable	Obs	Mean	Std. Dev.	Min	Max
id	241	900.1701	484.3652	5	1717