SC705: Advanced Statistics
Instructor: Natasha Sarkisian
Class notes: HLM Model Building Strategies

Model Selection Strategy

To summarize, we saw that multilevel models can include 3 types of predictors:

- Level-1 predictors (e.g., student SES)
- Level-2 predictors (e.g., school SECTOR)
- Level-1 predictors aggregated to level 2 (e.g., MEANSES)

In addition, we have a number of choices:

- The intercept can be estimated as either fixed or random (typically random)
- The effects of level 1 predictors can be estimated as either fixed effects or random effects
- Level 2 predictors can be used to predict the intercept (i.e., as direct predictors of DV)
- Level 2 predictors can explain the variation in slopes of level 1 predictors (i.e., as cross-level interactions)

Because so many components are involved, it is best to proceed incrementally. Two main algorithms are recommended; the first one differentiates between level 1 and level 2 variables; the second one does not.

Level-specific algorithm:

1. Fit a fully unconditional model (Model 0). Evaluate level 2 variance to see if HLM is necessary.
2. Estimate a model with random intercept and slopes using only level 1 variables (Model 2) and any necessary interactions among them. Make all slopes random, unless you have substantive reasons for separating random and non-random ones. Note, however, that random slopes for interaction terms can be difficult to interpret.
3. Evaluate slope variance, decide whether some slopes should be non-random, and fix those slopes. (Do a joint significance test to doublecheck that all those slopes are jointly not significant.)
4. Based on the significance of regression coefficients, exclude variables where both coefficients and corresponding random effects are not significant. Keep the variable if the coefficient is non-significant but the random effect is. Make sure to conduct hypotheses tests to make sure these variables are jointly not significant. (Note that sometimes you might have substantive reasons to keep the variable even if its coefficient is not significant.)
5. Estimate means-as-outcomes with level 1 covariates model (Model 4) to select level 2 predictors of intercept (include both original level 2 variables and aggregates of level 1). Use hypothesis testing to trim the model.
6. For slopes with significant variance, use level 2 predictors to explain that variance (i.e., estimate an intercepts-and-slopes-as-outcomes model – Model 5). If a slope does not have significant variance but your theory suggests cross-level interaction, do include such an interaction. Use hypothesis testing to trim the model.

7. If the slope variance remaining after entering level 2 predictors is not statistically significant, estimate that slope as non-randomly varying (Model 6).

Combined algorithm:

1. Fit a fully unconditional model (Model 0). Evaluate level 2 variance to see if HLM is necessary.
2. Enter all level 2 and level 1 variables in the model, and include any within-level and cross-level interactions based on theory (Model 5). (Don't forget to use aggregates of level 1 variables.) Make all slopes random, unless you have substantive reasons for separating random and non-random ones. Note, however, that random slopes for interaction terms can be difficult to interpret.
3. Evaluate slope variance, decide whether some slopes should be non-random, and fix those slopes. (Do a joint significance test to doublecheck that all those slopes are jointly not significant.)
4. Based on the significance of regression coefficients, exclude variables where both coefficients and corresponding random effects are not significant. Keep the variable if the coefficient is non-significant but the random effect is. Make sure to conduct hypotheses tests to make sure these variables are jointly not significant. (Note that sometimes you might have substantive reasons to keep the variable even if its coefficient is not significant.)
5. If there are remaining random slopes with significant variance, consider adding other cross-level interactions to explain that variance. If that leads to the random slope becoming non-significant, estimate that slope as non-randomly varying (Model 6).

Using Hypothesis Testing to Build Models

When making decisions what variables to include and whether to estimate random or fixed effects, we need to use hypothesis testing tools. HLM6 allows you to test various hypotheses which can be helpful when evaluating which variables and which random effects to include in your model. The basic idea behind hypothesis testing is to build a set of contrasts that would add up to zero under the null hypothesis, and then test the hypothesis that, combined, they are indeed zero.

1. Single parameter tests of significance.

Single parameter tests are presented in your regular HLM output; in practice, there is no need to run such tests in addition to the regular output, but for learning purposes, we will start with these. Suppose we want to test whether a specific coefficient (e.g. the SES slope for average SES public schools (i.e., intercept for SES slope, gamma 10) is zero. The set of contrasts that we will specify for that will include 1 for γ_{10} and 0 for everything else; therefore, we will test Ho: γ_{10}=0.

Level-1 Model

$$MATHACH_{ij} = \beta_{0j} + \beta_{1j}*(SES_{ij}) + r_{ij}$$

Level-2 Model

$\beta_{0j} = \gamma_{00} + \gamma_{01}*(SECTOR_j) + \gamma_{02}*(MEANSES_j) + u_{0j}$
$\beta_{1j} = \gamma_{10} + \gamma_{11}*(SECTOR_j) + \gamma_{12}*(MEANSES_j) + u_{1j}$

Mixed Model

$MATHACH_{ij} = \gamma_{00} + \gamma_{01}*SECTOR_j + \gamma_{02}*MEANSES_j$
$+ \gamma_{10}*SES_{ij} + \gamma_{11}*SECTOR_j*SES_{ij} + \gamma_{12}*MEANSES_j*SES_{ij}$
$+ u_{0j} + u_{1j}*SES_{ij} + r_{ij}$

Final Results - Iteration 213

Iterations stopped due to small change in likelihood function

$\sigma^2 = 36.74002$

τ

INTRCPT1, β_0	2.41161	0.19250
SES, β_1	0.19250	0.05740

τ (as correlations)

INTRCPT1, β_0	1.000	0.517
SES, β_1	0.517	1.000

Random level-1 coefficient	Reliability estimate
INTRCPT1, β_0	0.670
SES, β_1	0.030

The value of the log-likelihood function at iteration 213 = -2.325183E+004

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.095921	0.202970	59.595	157	<0.001
SECTOR, γ_{01}	1.193603	0.308405	3.870	157	<0.001
MEANSES, γ_{02}	3.326678	0.389264	8.546	157	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.904442	0.150182	19.340	157	<0.001
SECTOR, γ_{11}	-1.576099	0.227461	-6.929	157	<0.001
MEANSES, γ_{12}	0.841643	0.275338	3.057	157	0.003

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast
For INTRCPT1, β_0		

INTRCPT2, γ_{00}	12.095921	0.0000
SECTOR, γ_{01}	1.193603	0.0000
MEANSES, γ_{02}	3.326678	0.0000
For SES slope, β_1		
INTRCPT2, γ_{10}	2.904442	1.0000
SECTOR, γ_{11}	-1.576099	0.0000
MEANSES, γ_{12}	0.841643	0.0000
Estimate		2.9044
Standard error of estimate		0.1502

χ^2 statistic = 374.016800
Degrees of freedom = 1
p-value = <0.001

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.095921	0.184221	65.660	157	<0.001
SECTOR, γ_{01}	1.193603	0.312200	3.823	157	<0.001
MEANSES, γ_{02}	3.326678	0.378898	8.780	157	<0.001
For SES slope, β_1					
INTRCPT2, γ_{10}	2.904442	0.142472	20.386	157	<0.001
SECTOR, γ_{11}	-1.576099	0.226476	-6.959	157	<0.001
MEANSES, γ_{12}	0.841643	0.299788	2.807	157	0.006

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast
For INTRCPT1, β_0		
INTRCPT2, γ_{00}	12.095921	0.0000
SECTOR, γ_{01}	1.193603	0.0000
MEANSES, γ_{02}	3.326678	0.0000
For SES slope, β_1		
INTRCPT2, γ_{10}	2.904442	1.0000
SECTOR, γ_{11}	-1.576099	0.0000
MEANSES, γ_{12}	0.841643	0.0000
Estimate		2.9044
Standard error of estimate		0.1425

χ^2 statistic = 415.593615
Degrees of freedom = 1
p-value = <0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	1.55293	2.41161	157	573.17259	<0.001
SES slope, u_1	0.23958	0.05740	157	162.63041	0.362
level-1, r	6.06135	36.74002			

Statistics for current covariance components model

Deviance = 46503.667345
Number of estimated parameters = 4

Here, we reject Ho based on both sets of results – with regular SE and with robust SE. So we cannot omit SES.

2. Multi-parameter tests of significance.
Here, we test the hypothesis that multiple coefficients are all equal to 0. Typically, we do that in order to decide whether they can be omitted from the model. This can either be coefficients for different variables (possibly related, e.g. sets of dummies), or coefficients for the same variable in different parts of the model. For example, for could test that all coefficients for SES slope are zero. That would mean testing a combined hypothesis:
$\gamma_{10}=0$
$\gamma_{11}=0$
$\gamma_{12}=0$
We can do that by selecting 1 for each of these coefficients when selecting contrasts (separate column for each; then run the model):

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast		
For INTRCPT1, β_0				
INTRCPT2, γ_{00}	12.095921	0.0000	0.0000	0.0000
SECTOR, γ_{01}	1.193603	0.0000	0.0000	0.0000
MEANSES, γ_{02}	3.326678	0.0000	0.0000	0.0000
For SES slope, β_1				
INTRCPT2, γ_{10}	2.904442	1.0000	0.0000	0.0000
SECTOR, γ_{11}	-1.576099	0.0000	1.0000	0.0000
MEANSES, γ_{12}	0.841643	0.0000	0.0000	1.0000
Estimate		2.9044	-1.5761	0.8416
Standard error of estimate		0.1425	0.2265	0.2998

χ^2 statistic = 510.445059
Degrees of freedom = 3
p-value = <0.001

These are robust SE results, and we reject Ho; the coefficients associated with SES slope are jointly significant. We can also test whether MEANSES is significant across equations:

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast	
For INTRCPT1, β_0			
INTRCPT2, γ_{00}	12.095921	0.0000	0.0000
SECTOR, γ_{01}	1.193603	0.0000	0.0000
MEANSES, γ_{02}	3.326678	1.0000	0.0000
For SES slope, β_1			
INTRCPT2, γ_{10}	2.904442	0.0000	0.0000
SECTOR, γ_{11}	-1.576099	0.0000	0.0000
MEANSES, γ_{12}	0.841643	0.0000	1.0000
Estimate		3.3267	0.8416
Standard error of estimate		0.3789	0.2998

χ^2 statistic = 77.290996
Degrees of freedom = 2
p-value = <0.001

This test is used to jointly test whether multiple variables have non-significant coefficients and therefore can be omitted (see step 4 of both model selection algorithms). Here, they cannot be omitted as we reject H_0.

3. Tests for equality of coefficients.

We can also test whether two or more coefficients are equal. This is typically used when we have a series of related dummy variables, and we want to combine some dummies. E.g., we could have students' racial/ethnic identification, with dummy variables representing African American, Mexican American, Puerto Rican, Asian American, etc., the omitted category being White. We could then wonder whether we could simplify that into one dichotomy, White vs ethnic minority. But to test whether the data support this simplification, we'd test whether coefficients for each ethnic group equal to each other (i.e., are all groups different from Whites in the same way?). We don't have sets of dummy variables in this dataset, so I will show how to do it with a less realistic example. Suppose we want to test if the effect of SECTOR equals the effect of MEANSES. Then we test:

$\gamma_{01} = \gamma_{02}$
$\gamma_{11} = \gamma_{12}$

To do this using contrasts, we redefine it as:

$\gamma_{01} - \gamma_{02} = 0$
$\gamma_{11} - \gamma_{12} = 0$

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast	
For INTRCPT1, β_0			
INTRCPT2, γ_{00}	12.095921	0.0000	0.0000
SECTOR, γ_{01}	1.193603	1.0000	0.0000
MEANSES, γ_{02}	3.326678	-1.0000	0.0000
For SES slope, β_1			
INTRCPT2, γ_{10}	2.904442	0.0000	0.0000
SECTOR, γ_{11}	-1.576099	0.0000	1.0000
MEANSES, γ_{12}	0.841643	0.0000	-1.0000
Estimate		-2.1331	-2.4177
Standard error of estimate		0.5585	0.4435

χ^2 statistic = 37.456012
Degrees of freedom = 2
p-value = <0.001

Here, we reject Ho, so we wouldn't be able to combine those variables (not that we really wanted to in this hypothetical example).

If we had a larger set of dummies we wanted to combine – say, three variables – we would contrast each to the first one in the list, so we would need 4 contrasts:
$\gamma_{01} = \gamma_{02}$
$\gamma_{01} = \gamma_{03}$
$\gamma_{11} = \gamma_{12}$
$\gamma_{11} = \gamma_{13}$
So there would be four columns with the following 1 and -1 values:
$1*\gamma_{01} + (-1)*\gamma_{02}=0$
$1*\gamma_{01} + (-1)*\gamma_{03}=0$
$1*\gamma_{11} + (-1)*\gamma_{12}=0$
$1*\gamma_{11} + (-1)*\gamma_{13}=0$

4. Tests for variance components

If we are interested in testing hypotheses about variance components or their combinations (e.g., see step 3 in both model-building algorithms), we should utilize likelihood ratio tests based on deviance values. To use such test, we estimate two models, and calculate D0-D1. The resulting difference follows chi-square distribution with df=number of parameters for model 0 minus number of parameters for model 1.

From the model presented above (where SES slope was random), we find:
Deviance = 46503.667345
Number of estimated parameters = 4

We change the model, making the SES slope non-varying, and go to Other Settings → Hypothesis Testing → Test against another model, and enter these deviance and number of parameters. Then we run the model; in the output, we see:

Level-1 Model

$MATHACH_{ij} = \beta_{0j} + \beta_{1j}*(SES_{ij}) + r_{ij}$

Level-2 Model

$\beta_{0j} = \gamma_{00} + \gamma_{01}*(SECTOR_j) + \gamma_{02}*(MEANSES_j) + u_{0j}$
$\beta_{1j} = \gamma_{10} + \gamma_{11}*(SECTOR_j) + \gamma_{12}*(MEANSES_j)$

Mixed Model

$MATHACH_{ij} = \gamma_{00} + \gamma_{01}*SECTOR_j + \gamma_{02}*MEANSES_j$
$+ \gamma_{10}*SES_{ij} + \gamma_{11}*SECTOR_j*SES_{ij} + \gamma_{12}*MEANSES_j*SES_{ij}$
$+ u_{0j}+ r_{ij}$

Statistics for current covariance components model

Deviance = 46504.376104
Number of estimated parameters = 2

Variance-Covariance components test
χ^2 statistic = 0.70876
Degrees of freedom = 2
p-value = >.500

P-value indicates that there is no significant difference in model fit between these two models – and if two models – one more complicated, and the other one simpler – are not significantly different, we should pick the simpler model (i.e., more parsimonious one). Of course, we already knew that SES slope is not significant based on the output for variance components. But if we have multiple slopes that we think should be fixed rather than random, we can do such a test for more than one variance component simultaneously by comparing a model with random slopes to that where these slopes are fixed – that is the main use of this test.

The issue of centering

You have already noticed that HLM6 asks you whether and how you'd like to center your predictors. Here, we will discuss the issues involved in making these decisions.

Level-1 predictors:

1. Natural metric (X):
You should only use the original metric if the value of 0 for a predictor is a meaningful value. When 0 is not meaningful, the estimate of the intercept will be arbitrary and may be estimated with poor precision. Lack of precision in HLM can be very problematic. First, because you are estimating within-group intercepts, thus with possibly small N, the estimates may be quite unstable. Second, because you may be trying to model variation in these intercepts, your model will be affected by the unreliability of the estimates.

2. Grand-mean centering (X - grand mean):

This will address the problems with estimation of intercept in original metric. Because the 0 values will fall in the middle of the distribution of the predictors, the intercept estimates will be estimated with much more precision. The intercept is also interpretable. Specifically, it will represent the group-mean value for a person with a (grand) average on every predictor. The interpretation of the intercepts is now "adjusted group mean." The interpretation of slopes does not change. E.g. our measure of SES is already grand-mean centered because it is a standardized scale. So we can interpret the fixed effect for the intercept as the average math achievement adjusted for SES – i.e., the average math achievement for someone with average SES.

LEVEL 1 MODEL

$$MATHACH_{ij} = \beta_{0j} + \beta_{1j}(FEMALE_{ij}) + \beta_{2j}(SES_{ij} - \overline{SES}_{..}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(SECTOR_j) + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}(SECTOR_j) + u_{1j}$$
$$\beta_{2j} = \gamma_{20} + \gamma_{21}(SECTOR_j)$$

```
   Level-1                  Level-2
   Coefficients             Predictors
 ----------------------   ---------------
         INTRCPT1, B0      INTRCPT2, G00
                             SECTOR, G01
     FEMALE slope, B1      INTRCPT2, G10
                             SECTOR, G11
#       SES slope, B2      INTRCPT2, G20
                             SECTOR, G21

'#' - The residual parameter variance for this level-1 coefficient has been set
      to zero.

 Summary of the model specified (in equation format)
 ---------------------------------------------------

Level-1 Model

       Y = B0 + B1*(FEMALE) + B2*(SES) + R

Level-2 Model
       B0 = G00 + G01*(SECTOR) + U0
       B1 = G10 + G11*(SECTOR) + U1
       B2 = G20 + G21*(SECTOR)

 Sigma_squared =     36.45566

 Tau
 INTRCPT1,B0      4.20999      -1.19503
   FEMALE,B1     -1.19503       1.10455


Tau (as correlations)
 INTRCPT1,B0  1.000 -0.554
   FEMALE,B1 -0.554  1.000

 ----------------------------------------------------
```

Random level-1 coefficient	Reliability estimate
INTRCPT1, B0	0.681
FEMALE, B1	0.236

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, B0					
INTRCPT2, G00	12.437528	0.253308	49.100	158	0.000
SECTOR, G01	2.084354	0.416840	5.000	158	0.000
For FEMALE slope, B1					
INTRCPT2, G10	-1.222739	0.220850	-5.537	158	0.000
SECTOR, G11	0.031679	0.401264	0.079	158	0.938
For SES slope, B2					
INTRCPT2, G20	2.919985	0.141351	20.658	7179	0.000
SECTOR, G21	-1.293137	0.207803	-6.223	7179	0.000

Final estimation of variance components:

Random Effect		Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1,	U0	2.05183	4.20999	121	347.17034	0.000
FEMALE slope,	U1	1.05098	1.10455	121	153.20942	0.025
level-1,	R	6.03785	36.45566			

Note that while it may seem inappropriate at first to center a dummy variable, in HLM it actually is quite useful. If the dummy is uncentered, the intercept is the average value when the dummy variable is 0. If the dummy variable is centered, the intercept then becomes the mean adjusted for the proportion of cases with the dummy variable=1. For example, if the indicator for sex variable is centered around the grand mean, this centered predictor can take two values. If the subject is female, it will equal the proportion of male students in the sample. If the subject is male, it will equal to minus the proportion of female students in the sample. Zero on this variable becomes the average proportion of female students. The intercept again will be the adjusted group mean – in this case, it is adjusted for the difference among level-2 units in the percentage of female students.

LEVEL 1 MODEL

$$MATHACH_{ij} = \beta_{0j} + \beta_{1j}(FEMALE_{ij} - \overline{FEMALE}_{..}) + \beta_{2j}(SES_{ij} - \overline{SES}_{..}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(SECTOR_j) + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}(SECTOR_j) + u_{1j}$$
$$\beta_{2j} = \gamma_{20} + \gamma_{21}(SECTOR_j)$$

Level-1 Coefficients	Level-2 Predictors
INTRCPT1, B0	INTRCPT2, G00
	SECTOR, G01

```
 %   FEMALE slope, B1      INTRCPT2, G10
                             SECTOR, G11
#       SES slope, B2      INTRCPT2, G20
                             SECTOR, G21

'#' - The residual parameter variance for this level-1 coefficient has been set
      to zero.
'%' - This level-1 predictor has been centered around its grand mean.

Level-1 Model

      Y = B0 + B1*(FEMALE) + B2*(SES) + R

Level-2 Model
      B0 = G00 + G01*(SECTOR) + U0
      B1 = G10 + G11*(SECTOR) + U1
      B2 = G20 + G21*(SECTOR)

 Sigma_squared =     36.45669

 Tau
 INTRCPT1,B0      3.25698      -0.61167
   FEMALE,B1     -0.61167       1.09435


Tau (as correlations)
 INTRCPT1,B0  1.000 -0.324
   FEMALE,B1 -0.324  1.000

 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                        0.774
    FEMALE, B1                        0.234
 ----------------------------------------------------

 Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                       Standard             Approx.
   Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
    INTRCPT2, G00          11.791658   0.214281    55.029       158    0.000
      SECTOR, G01           2.101157   0.333510     6.300       158    0.000
 For   FEMALE slope, B1
    INTRCPT2, G10          -1.222670   0.220851    -5.536       158    0.000
      SECTOR, G11           0.032271   0.401209     0.080       158    0.936
 For      SES slope, B2
    INTRCPT2, G20           2.919869   0.141361    20.655      7179    0.000
      SECTOR, G21          -1.292989   0.207806    -6.222      7179    0.000
 ----------------------------------------------------------------------------

 Final estimation of variance components:
 -----------------------------------------------------------------------------
 Random Effect           Standard      Variance     df    Chi-square  P-value
                         Deviation     Component
 -----------------------------------------------------------------------------
 INTRCPT1,       U0        1.80471       3.25698   121     488.52692    0.000
   FEMALE slope, U1        1.04611       1.09435   121     153.19922    0.025
  level-1,       R         6.03794      36.45669
 -----------------------------------------------------------------------------
```

3. Group-mean centering (X – group mean):

Predictors can also be centered around the mean value for the group to which they belong. The intercept can then be interpreted as the average outcome for each group. This allows interpretation of parameter estimates as person-level effects within each group (i.e. if you differ from your group's average by one unit, your math achievement will increase by X units).

Again, we can group-mean center dummy variables as well. For females, we will get a value equal to the proportion of male students in school j; for males, it will take the value equal to minus the proportion of females in that school. The fact that it is a dummy variable does not change the interpretation of the intercept when group mean-centering is employed.

LEVEL 1 MODEL

$$MATHACH_{ij} = \beta_{0j} + \beta_{1j}(FEMALE_{ij} - \overline{FEMALE}_{\bullet j}) + \beta_{2j}(SES_{ij} - \overline{SES}_{\bullet j}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(SECTOR_j) + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}(SECTOR_j) + u_{1j}$$
$$\beta_{2j} = \gamma_{20} + \gamma_{21}(SECTOR_j)$$

```
 The outcome variable is  MATHACH
 The model specified for the fixed effects was:
 ----------------------------------------------------
   Level-1                  Level-2
   Coefficients             Predictors
 ----------------------   ---------------
         INTRCPT1, B0      INTRCPT2, G00
                             SECTOR, G01
 *   FEMALE slope, B1      INTRCPT2, G10
                             SECTOR, G11
#*      SES slope, B2      INTRCPT2, G20
                             SECTOR, G21

'#' - The residual parameter variance for this level-1 coefficient has been set
      to zero.
'*' - This level-1 predictor has been centered around its group mean.

Summary of the model specified (in equation format)
 ---------------------------------------------------
Level-1 Model

       Y = B0 + B1*(FEMALE) + B2*(SES) + R
Level-2 Model
       B0 = G00 + G01*(SECTOR) + U0
       B1 = G10 + G11*(SECTOR) + U1
       B2 = G20 + G21*(SECTOR)

 Sigma_squared =     36.45732

 Tau
 INTRCPT1,B0      6.75745      -0.63530
   FEMALE,B1     -0.63530       0.82580


Tau (as correlations)
```

```
 INTRCPT1,B0  1.000 -0.269
   FEMALE,B1 -0.269  1.000

 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                        0.882
    FEMALE, B1                        0.188
 ----------------------------------------------------
Note: The reliability estimates reported above are based on only 123 of 160
units that had sufficient data for computation.  Fixed effects and variance
components are based on all the data.

The value of the likelihood function at iteration 31 = -2.330178E+004

The outcome variable is  MATHACH
 Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                    Standard             Approx.
    Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
    INTRCPT2, G00          11.393469   0.292627    38.935       158    0.000
      SECTOR, G01           2.804207   0.436272     6.428       158    0.000
 For   FEMALE slope, B1
    INTRCPT2, G10          -1.224963   0.218270    -5.612       158    0.000
      SECTOR, G11           0.421184   0.422651     0.997       158    0.321
 For      SES slope, B2
    INTRCPT2, G20           2.732981   0.156703    17.440      7179    0.000
      SECTOR, G21          -1.310898   0.229605    -5.709      7179    0.000
 ----------------------------------------------------------------------------

 Final estimation of variance components:
 -----------------------------------------------------------------------------
 Random Effect           Standard      Variance     df    Chi-square  P-value
                         Deviation     Component
 -----------------------------------------------------------------------------
 INTRCPT1,       U0        2.59951       6.75745   121     890.99031    0.000
   FEMALE slope, U1        0.90873       0.82580   121     150.58868    0.035
  level-1,       R         6.03799      36.45732
 -----------------------------------------------------------------------------
Note: The chi-square statistics reported above are based on only 123 of 160
units that had sufficient data for computation.  Fixed effects and variance
components are based on all the data.
```

<u>Important:</u>
Under grand-mean centering or no centering, the parameter estimates reflect a combination of (1) person-level effects and (2) compositional effects. But when we use a group-centered predictor, we only estimate the person-level effects.

In order not to discard the compositional effects with group-mean centering, level-2 variables should be created to represent the group mean values for each group-mean centered predictor. Because the group mean is effectively removed from the individual scores, the level-2 values will be orthogonal to the level-1 values. Note that while HLM software has an option for group-mean centering, it does not compute the group mean values of a predictor to be included as a level-2 variable – you have to do that in another statistical package and then import the data into HLM (more on this below).

E.g. we can use group mean centering for SES and using mean SES as a school level variable (here, MEANSES is already in the dataset):

LEVEL 1 MODEL

$$MATHACH_{ij} = \beta_{0j} + \beta_{1j}(SES_{ij} - \overline{SES}_{\cdot\cdot}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(SECTOR_j) + \gamma_{02}(MEANSES_j) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(SECTOR_j) + \gamma_{12}(MEANSES_j) + u_{1j}$$

```
---------------------------------------------------

   Level-1                  Level-2
   Coefficients             Predictors
 ----------------------   ---------------
        INTRCPT1, B0      INTRCPT2, G00
                            SECTOR, G01
                           MEANSES, G02
 *      SES slope, B1     INTRCPT2, G10
                            SECTOR, G11
                           MEANSES, G12

'*' - This level-1 predictor has been centered around its group mean.
 Summary of the model specified (in equation format)
 ---------------------------------------------------

Level-1 Model

      Y = B0 + B1*(SES) + R

Level-2 Model
      B0 = G00 + G01*(SECTOR) + G02*(MEANSES) + U0
      B1 = G10 + G11*(SECTOR) + G12*(MEANSES) + U1

Sigma_squared =     36.70313

 Tau
 INTRCPT1,B0      2.37996       0.19058
      SES,B1      0.19058       0.14892


Tau (as correlations)
 INTRCPT1,B0  1.000  0.320
      SES,B1  0.320  1.000

 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                        0.733
       SES, B1                        0.073
 ----------------------------------------------------

 Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                    Standard             Approx.
    Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
```

```
For       INTRCPT1, B0
   INTRCPT2, G00         12.096006   0.173699    69.638       157    0.000
     SECTOR, G01          1.226384   0.308484     3.976       157    0.000
    MEANSES, G02          5.333056   0.334600    15.939       157    0.000
For      SES slope, B1
   INTRCPT2, G10          2.937981   0.147620    19.902       157    0.000
     SECTOR, G11         -1.640954   0.237401    -6.912       157    0.000
    MEANSES, G12          1.034427   0.332785     3.108       157    0.003
----------------------------------------------------------------------------

Final estimation of variance components:
-----------------------------------------------------------------------------
Random Effect           Standard      Variance     df    Chi-square  P-value
                        Deviation     Component
-----------------------------------------------------------------------------
INTRCPT1,       U0        1.54271       2.37996   157     605.29503    0.000
     SES slope, U1        0.38590       0.14892   157     162.30867    0.369
 level-1,       R         6.05831      36.70313
-----------------------------------------------------------------------------
```

Here, the effects of SES turn out to be quite complex: For those who are in a public school whose SES is at their school's average and whose school itself is average in terms of its SES, the math achievement is 12.096. If you are in a Catholic school with such properties, it's 12.1+1.2=13.3. But if your school's average SES is 1 unit higher that the average for all schools, then your math achievement increases by 5.33. Further, in addition to these school-level effects, your individual SES also plays a role – if you are in an average (in terms of SES) public school, one unit increase in your SES will raise your math score by 2.94. In a Catholic school, that effect would be 2.94-1.64=1.30. But if you are in a public school and your school is 1 unit above an average school in its SES, then your personal SES impact (per one unit) would be 2.94+1.03=3.97. For a Catholic school in that situation, that effect of SES would become 2.94-1.64+1.03=2.33. Interestingly, personal SES seems to have stronger impact on math achievement in those schools that have relatively high school-level SES.

The choice between grand-mean centering and group-mean centering depends on your theoretical thinking about processes. If you think that the absolute values of level 1 variable matter, then use grand-mean centering. If you think that it is the relative position of the person with regards to their group's mean is what matters, then use group-centering.

Level-2 predictors:

Centering issues for level-2 predictors are essentially the same issues faced in any regression. If the value of 0 for a predictor is not meaningful, the intercept will not have a meaningful interpretation and the estimate may lack precision. When these conditions exist, centering is advisable. You can either use grand-mean centering (then the intercept will reflect the average group) or center around some constant (then the intercept will reflect a group with the value of the predictor equal to that constant). Note that HLM6 only has the grand-mean centering option for level-2 predictors – if you want to center around some other value, you would have to generate such a centered variable in another statistical program and then import the data into HLM. Typically, however, grand-mean centering is just fine.

LEVEL 1 MODEL

$$MATHACH_{ij} = \beta_{0j} + \beta_{1j}(SES_{ij} - \overline{SES}_{..}) + r_{ij}$$

LEVEL 2 MODEL

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(SECTOR_j - \overline{SECTOR}_{.}) + \gamma_{02}(MEANSES_j - \overline{MEANSES}_{.}) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}(SECTOR_j - \overline{SECTOR}_{.}) + \gamma_{12}(MEANSES_j - \overline{MEANSES}_{.}) + u_{1j}$$

```
   Level-1                   Level-2
   Coefficients              Predictors
 ----------------------   ---------------
         INTRCPT1, B0      INTRCPT2, G00
$                            SECTOR, G01
$                           MEANSES, G02
 *      SES slope, B1      INTRCPT2, G10
$                            SECTOR, G11
$                           MEANSES, G12

'*' - This level-1 predictor has been centered around its group mean.
'$' - This level-2 predictor has been centered around its grand mean.

Summary of the model specified (in equation format)
 ---------------------------------------------------

Level-1 Model

       Y = B0 + B1*(SES) + R

Level-2 Model
       B0 = G00 + G01*(SECTOR) + G02*(MEANSES) + U0
       B1 = G10 + G11*(SECTOR) + G12*(MEANSES) + U1

 Sigma_squared =     36.70313

 Tau
 INTRCPT1,B0      2.37996       0.19058
      SES,B1      0.19058       0.14892


Tau (as correlations)
 INTRCPT1,B0  1.000  0.320
      SES,B1  0.320  1.000

 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                        0.733
       SES, B1                        0.073
 ----------------------------------------------------

 Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                      Standard             Approx.
    Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
```

```
   INTRCPT2, G00          12.631549   0.140082    90.173       157    0.000
     SECTOR, G01           1.226384   0.308484     3.976       157    0.000
    MEANSES, G02           5.333056   0.334600    15.939       157    0.000
For      SES slope, B1
   INTRCPT2, G10           2.219870   0.108224    20.512       157    0.000
     SECTOR, G11          -1.640954   0.237401    -6.912       157    0.000
    MEANSES, G12           1.034427   0.332785     3.108       157    0.003
----------------------------------------------------------------------------

Final estimation of variance components:
-----------------------------------------------------------------------------
Random Effect           Standard      Variance     df    Chi-square  P-value
                        Deviation     Component
-----------------------------------------------------------------------------
INTRCPT1,       U0        1.54271       2.37996   157     605.29503    0.000
     SES slope, U1        0.38590       0.14892   157     162.30867    0.369
 level-1,       R         6.05831      36.70313
-----------------------------------------------------------------------------
```

Creating Aggregated Variables from Level 1 Data

In Stata:
You can either use level 1 data file or (if you already have it) a single file for two levels. First sort this file by school id:

```
. use "C:\Documents and Settings\SARKISIN\My Documents\hsb1.dta", clear
```

Use egen command to generate an aggregated variable:

```
. bysort id: egen meanses2=mean(ses)
```

If you have two separate files, you'll end up generating this variable in level 1 file, and then you'll have to create a combined file for two levels by merging the two files:

```
. merge m:1 id using "C:\Documents and Settings\SARKISIN\My Documents\hsb2.dta"

    Result                           # of obs.
    -----------------------------------------
    not matched                             0
    matched                             7,185  (_merge==3)
    -----------------------------------------

. drop _merge
```

In SPSS:
Here you can use command Aggregate to generate a new aggregated variable in level-1 file or a merged file. If you have two separate files, you'll end up generating that variable in level 1 file, and you'll have to transfer it into level 2 (utilizing Merge function in SPSS).

```
AGGREGATE
 / BREAK = id
 / meanses2 = MEAN(ses).
```

Then merge:

```
MATCH FILES /FILE=*
 /TABLE='C:\ HSB2.SAV'
 /BY id.
EXECUTE.
```